Nonlinear evolution of a relativistically strong electromagnetic wave in the self-created electron-positron plasma

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The nonlinear interaction between the electron-positron pairs produced by an electromagnetic wave in plasma and the wave leads to damping of the wave, frequency upshift, change of polarization and particle acceleration. The case of a circularly polarized wave is investigated in the framework of the relativistic Vlasov equation with a source term based on the Schwinger formula for the pair creation rate.

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The effect of particle creation in vacuum by an external field was first used to investigate the production of electron positron pairs in a constant, spatially homogeneous electric field [1 – 3]. It is often referred to as the Schwinger mechanism and is considered as one of the most intriguing nonlinear phenomena in quantum field theory. The effect lies beyond the reach of the perturbation theory and its experimental verification would test the validity of the theory in the region of strong fields. It is believed that the most probable way of detecting this effect is connected with e^+e^- pair production under the action of a time-varying electromagnetic (EM) field [4 – 7], that are produced by lasers, especially, in view of recent developments of laser technology [8]. In addition, several methods for reaching the critical intensity, $I_{Sch} = 4.6 \cdot 10^{29}$ W/cm^2, which corresponds, for a laser pulse with wavelength $\approx 1\mu$m, to an electric field equal to the critical Schwinger field $E_{Sch} = 1.32 \cdot 10^{16}$ V/cm, with presently available systems have been proposed recently [9, 10].

However we should note that in Refs. [1 – 7] the mutual interaction of the particles produced and the effect of these particles on the original electric field (backreaction) are not taken into account. The problem of the backreaction of the produced particles on the background field was discussed extensively in a number of papers on the particle formation process in high energy hadronic interactions [11 – 13] as well as under the action of electric fields [14]. It was understood that, in solving a dynamical problem with a strong initial electric field, the effect of the produced particles on the electric field (the back reaction) should be taken into consideration. A kinetic equation coupled to Maxwell equations was used to solve this problem. However the spatially homogeneous time dependent electric field, which was used, is not a solution of Maxwell equations in vacuum. We also note that in Refs. [11 – 14] special attention was paid to the properties of the emerging plasma, while the properties of the background field were not studied in detail.

In the present letter, we consider the process of e^+e^- pair production in a cold collisionless plasma by an EM field which is an actual solution of the Maxwell equations (the similar approach was used in Refs. [15, 16]), as well as the backreaction of the produced pairs on the background field. In doing so we use the kinetic equation, with a source term obtained from the pair production rate [11, 12]. In order to elucidate the role of the magnetic field component of the EM wave on the e^+e^- pair production, we consider a planar, circularly polarized wave propagating in an underdense collisionless plasma [2)] (for the sake of simplicity we consider an e^+e^- plasma):

$$\mathbf{A} = A_0 \left[\mathbf{e}_x \cos(\omega t' - kz) + \mathbf{e}_y \sin(\omega t' - kz)\right], \qquad (1)$$

where A_0, ω, and k are wave amplitude, frequency, and wave vector respectively, t' is the time in the laboratory frame. In the case of a planar wave in a plasma the first field invariant $\mathcal{F}$ is not equal to zero due to the different dispersion equation with respect to that in vacuum:

$$\mathcal{F} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) = \frac{\Omega^2}{2} A_0^2 \equiv \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2 E_0^2, \qquad (2)$$

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[2)]In the following we use the $c = 1$ and $\hbar = 1$ convention.

where $\Omega = \left[8\pi e^2 n_0/(m^2+P^2)^{1/2}\right]^{1/2}$ is the Langmuir frequency which enters the dispersion equation for the EM wave propagating in plasma in the laboratory frame:

$$\omega^2 = k^2 + \Omega^2, \tag{3}$$

and P is the momentum of plasma particles. Therefore, in a plasma, e^+e^- pairs can be produced by a planar EM wave, as was shown in Ref.[15]. The nonlinear EM wave in plasmas is also characterized by the dependence of its phase and group velocity on the plasma parameters and on the wave amplitude ($v_{ph} = \omega/k$, $v_g = \partial\omega/\partial k$, see Eq. (3)). In this case a Lorentz transform to the reference frame moving with the group velocity v_g of the wave transforms the EM field into a purely electric field, that rotates with constant frequency, and with no associated magnetic field

$$\mathbf{E} = \Omega A_0 \left(\mathbf{e}_x \sin\Omega t - \mathbf{e}_y \cos\Omega t\right). \tag{4}$$

Further we shall use the notation $E = \Omega A_0 \equiv (\Omega/\omega)E_0$. Although this transformation reduces the problem under consideration to the situation where the pairs are produced by a time-varying electric field, the effects of the wave magnetic field are incorporated rigorously into our model. See also Refs. [17, 18].

We consider the propagation of a circularly polarized EM wave in an underdense collisionless plasma in the boosted frame of reference. The relativistic kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + e_\alpha \mathbf{E}(t)\frac{\partial f_\alpha}{\partial \mathbf{p}} = q_\alpha(|\mathbf{E}|, p), \tag{5}$$

describes the dependence on time and momentum of the positron(electron) distribution function $f_\alpha(\mathbf{p},t)$ in the boosted frame where a spatially homogeneous electric field $\mathbf{E}(t)$ is present. The distribution function is normalized in such way that $\int f_\alpha(\mathbf{p},t)d^3p/(2\pi)^3 = n_\alpha$ gives the number n_α of positrons or electrons per unit volume, and e_α is their electric charge with $\alpha = +$ for positrons and $\alpha = -$ for electrons. The source term in Eq.(5) is proportional to the quasiclassical probability

$$\exp\left[-\frac{\pi(m^2+p_\perp^2)}{|e\mathbf{E}(t)|}\right] \tag{6}$$

of tunneling through the gap between the lower and the upper continuum of electron energy spectrum in the presence of the constant electric field. However, the naive estimation of characteristic time of pair production c/l_c, $l_c = \hbar/mc$, as well as the quasiclassical tunneling time $t_{tun} = 1/a\omega$ [6, 15], where $a = eA/mc$, is negligible with respect to the wave period. Thus, it is possible to use expression (6) for the time-varying electric field with time playing the role of a parameter. In addition, following the reasoning of Refs. [11, 12], we assume that the pairs are produced at rest, i. e. the momentum distribution of the source term is taken to be proportional to the Dirac delta function

$$q_\alpha(|\mathbf{E}|, p) = 2e^2|\mathbf{E}(t)|^2 \exp\left[-\frac{\pi m^2}{|e\mathbf{E}(t)|}\right]\delta(\mathbf{p}). \tag{7}$$

This assumption is reinforced by the fact that the momentum distribution in Eq.(6) has a width $p_\perp \sim \sim (|e\mathbf{E}(t)|)^{1/2} = m(|\mathbf{e}(t)|)^{1/2} \ll m$ which is negligible with respect to the momentum that electrons (positrons) acquire in the electric field. Here $\mathbf{e} = \mathbf{E}/E_{Sch} \ll 1$ is the normalized electric field and $E_{sch} = m^2/e$ is the critical Schwinger field. The source term has been normalized in such way that

$$W(\mathbf{E}) = \int q_\alpha(|\mathbf{E}|, \mathbf{p})\frac{d^3p}{(2\pi)^3} = \frac{|e\mathbf{E}|^2}{4\pi^3}\exp\left[-\frac{\pi m^2}{|e\mathbf{E}|}\right], \tag{8}$$

where $W(E)$ gives of positrons (electrons) produced according to Schwinger's formula.

We solve Eq.(5) by integrating it along the particle characteristics. Introducing a function $\mathbf{A}(t) = = -\int_0^t \mathbf{E}(s)ds$, we obtain the distribution function

$$\begin{aligned} f_\alpha = {} & f_{\alpha,0}[p_\parallel, \mathbf{p}_\perp + e_\alpha \mathbf{A}(t)] + \\ & + \int_0^t q_\alpha\{\mathbf{p}_\perp + e_\alpha[\mathbf{A}(t) - \mathbf{A}(s)], s\}ds, \end{aligned} \tag{9}$$

where $f_{\alpha,0}(p_\parallel, \mathbf{p}_\perp)$ is the distribution function of the initial plasma positrons (electrons) before the passage of the EM wave. Let us assume that at the initial time $t = 0$ the plasma is cold so that $f_{\alpha,0} = n_0(2\pi)^3\delta(p_\perp)\delta(p_\parallel - - p_{\parallel,0})$, where $p_\perp$ and $p_\parallel$ are the components of the particle momentum perpendicular and parallel to the direction of EM wave propagation and $p_{\parallel,0}$ is its initial value which arises from the Lorentz transform from the laboratory to the boosted frame.

The modification of the kinetic equation given by the source term in Eq.(7) must also be accompanied by a change of the source term in Maxwell equations. The e^+e^- pair production leads to the appearance of a time-dependent electric dipole which generates a polarization current. Thus the current density in Maxwell equation for the electric field acquires an additional term with respect to the situation when no pair production is present [12]:

$$\frac{d\mathbf{E}}{dt} = -4\pi\mathbf{j}_{tot} = -4\pi\left(\mathbf{j}_{cond} + \mathbf{j}_{pol}\right). \tag{10}$$

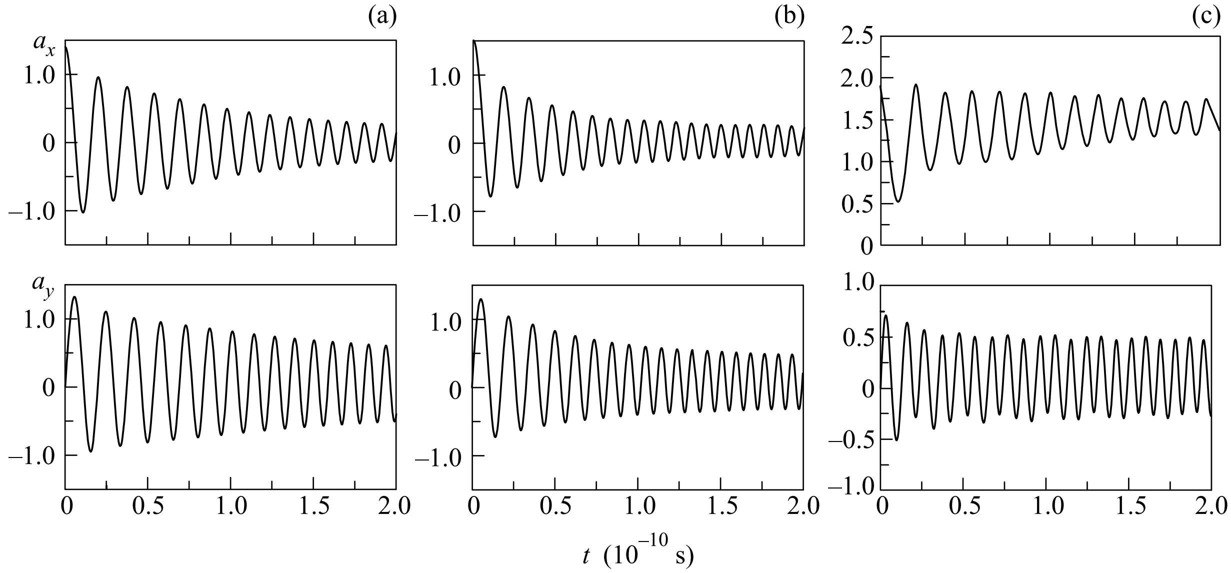

Fig.1. Time evolution in the moving frame of the x and the y-components of the dimensionless vector potential for different initial amplitudes: $a = 1.4 \cdot 10^5$ (a), $a = 1.5 \cdot 10^5$ (b), $a = 1.9 \cdot 10^5$ (c) with initial plasma density $n_0 = 10^{19}\,\mathrm{cm}^{-3}$ in the moving frame; $v_g \approx 1$, $\gamma_g = 10$. The upper row shows the x-component of the vector-potential, the lower the y-component. On the x-axis time is measured in seconds; $a = 1$ corresponds, for a 1 μm wavelength pulse, to an intensity of 10^{18} W/cm^2 and $a = 4.6 \cdot 10^5$ to the Schwinger intensity

Here the conduction and polarization [11] currents are

$$\mathbf{j}_{cond}(t) = e \sum_{\alpha=+,-} \int f_\alpha(\mathbf{p},t) \frac{\mathbf{p}}{\mathcal{E}} \frac{d^3p}{(2\pi)^3},$$
$$\mathbf{j}_{pol}(t) = \frac{\mathbf{E}(t)}{|\mathbf{E}(t)|^2} \sum_{\alpha=+,-} \int q_\alpha(\mathbf{p},t) \mathcal{E} \frac{d^3p}{(2\pi)^3}, \tag{11}$$

where $\mathcal{E} = (m^2+p^2)^{1/2}$. Using the distribution function (9), the dimensionless vector-potential $\mathbf{a} = e\mathbf{A}/m$ and the normalized electric field $\mathbf{e} = e\mathbf{E}/m^2$, we obtain the system of equations for the electric field evolution in the presence of pair production

$$\frac{d\mathbf{a}(t)}{dt} = -m\mathbf{e}(t),$$
$$\frac{d\mathbf{e}(t)}{dt} = \frac{\Omega^2}{m}\mathbf{a}(t) - \frac{em}{2\pi^2}\mathbf{e}(t) \exp\left[-\frac{\pi}{|\mathbf{e}(t)|}\right] + \tag{12}$$
$$+ \frac{\kappa}{8\pi^3 m^2} \int_0^t \frac{[\mathbf{a}(t) - \mathbf{a}(s)]|\mathbf{e}(s)|^2}{[1+|\mathbf{a}(t)-\mathbf{a}(s)|^2]^{1/2}} \exp\left[-\frac{\pi}{|\mathbf{e}(s)|}\right] ds.$$

Here $P = m[1+\tilde{p}_{\|0}^2+a^2(t)]^{1/2}$ (see definition of Ω, given above), $\tilde{p}_{\|0} \equiv p_{\|0}/m$ and $\kappa = 8\pi e^2 m^4$, where the factor m^4 stands for the inverse of the invariant Compton 4-volume $m^4 = c/l_c^4 \approx 0.14 \cdot 10^{53}\ \mathrm{cm}^{-3}\cdot \mathrm{s}^{-1}$.

Numerical solutions of this system are presented in Fig.1 for different initial amplitudes. We can see that the process of e^+e^- pair production leads to the damping of the wave in the plasma and to the nonlinear up-shift of its frequency. The damping occurs due to the fact that each event of the pair creation takes a portion of the field energy equal to $2mc^2$ and the amount needed for the particle acceleration. The up-shift of the field frequency is due to the increase of the plasma density, and thus of the Langmuir frequency, as new pairs are created, see also Ref. [18]. This frequency up-shift is seen in Fig.1, and bears a strong resemblance to the blue-shift of an EM wave that propagates in a medium that becomes ionized, as investigated theoretically in Ref.[19] and experimentally in Ref.[20].

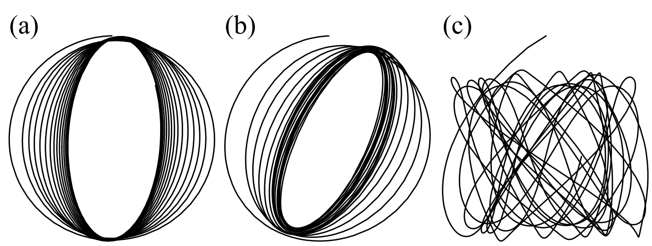

Fig.2. Trajectories of the projections of the electric field polarization vector for the same set of initial conditions as in Fig.1

Since the pair production rate depends on the field amplitude exponentially, an unbalanced damping of the field components can occur and lead to a change of the

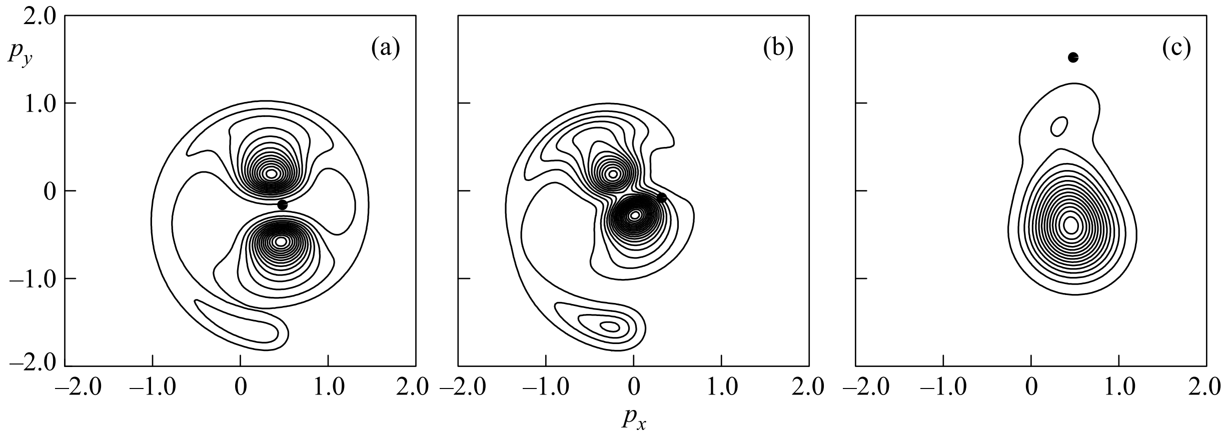

Fig.3. The electron distribution functions versus p_x and p_y in the moving reference frame for the same set of initial conditions as in Fig. 1 at time $2 \cdot 10^{-10}$ s. Particle momenta are normalized on the dimensionless vector-potential amplitude a multiplied times 10^5. Black circles correspond to the initial plasma particle distribution at time $2 \cdot 10^{-10}$ s

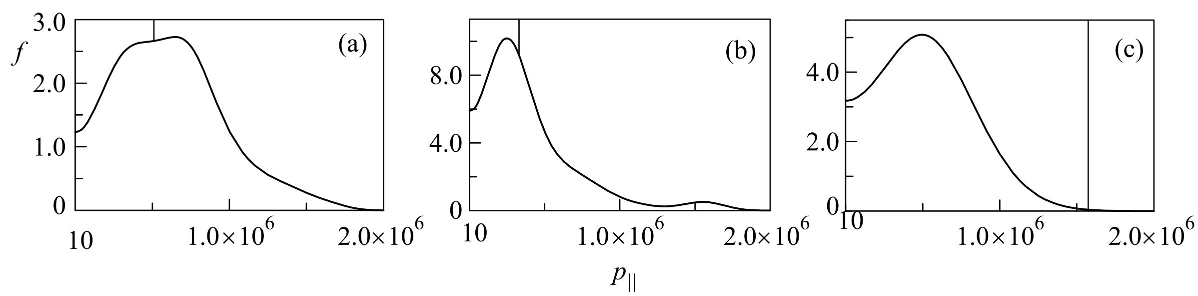

Fig.4. The electron distribution functions versus $p_{||}$ in the laboratory reference frame for the same set of initial conditions as in Fig.1 at time $2 \cdot 10^{-10}$ s. Momentum is measured in the same units as in Fig.3. The vertical lines correspond to the distribution of the initial plasma particles

field polarization. These properties of the electric field are shown in Fig.2, where the projections of the polarization vector are presented for the same set of initial parameters as in Fig.1. In Fig.2a we see the damping of the x-component of the electric field and the transition from circular to elliptic polarization with the major axis of the ellipse directed along the y-axis. In addition, in Fig.2b we see a rotation of the principal axes of the ellipse. The situation shown in Fig.2c is different from previous. In this latter case the pair production rate at the beginning of the field evolution is so large that the first wave oscillation cycle cannot be completed, leading to oscillations of the x-component of the wave vector potential around a non-zero mean value determined by the balance between the time averaged parts of the first two terms on the r.h.s. of the second of Eqs. (12). This shift of the center of the oscillations of the x-component of the vector potential leads to a reduction of the oscillation frequency of this wave component so that, in this case, the x and the y-components of the wave oscillate at different frequencies.

The difference between the above three cases is clearly illustrated by the different shapes of the particle distribution functions in the p_x–p_y plane (Note that the electron and the positron distributions are one the mirror image of the other). In cases a) and b) electrons and positrons are mostly created at the maxima of the electric field $|\mathbf{E}|$ (and thus of the vector potential $|\mathbf{A}|$). Since at birth $\mathbf{p}_\perp = 0$, in the case of a circularly polarized electric field this should lead to a ring type distribution. However, since the wave polarization becomes elliptical because of the backreaction due to the pair creation, the distribution function of each population consists, in the canonical momentum $\mathbf{p}_\perp + e_a\mathbf{A}$ plane, mainly of two blobs at $\pm e_a\mathbf{A}_{\max}$. In the p_x-p_y plane, these blobs move according to the time evolution of the vector potential $\mathbf{A}$. On the contrary the position of the initial distribution function (denoted by a dark dot in the figure) corresponds to $\mathbf{p}_\perp + e_a\mathbf{A} = 0$. In case c) the pairs are created mostly at the start at $\mathbf{p}_\perp + e_a\mathbf{A} = e_a\mathbf{A}(t=0)$. Since the time evolution of $\mathbf{A}(t)$ is ergodic, as shown in Fig.3, their distribution tends to be randomized in the p_x-p_y plane.

The particle distribution function is shown in Fig.4 versus the parallel momentum $p_{||}$ in the laboratory frame. Note that in case c) the strong damping of the wave due to the pair creation and the resulting non adiabatic interaction has led to a strong acceleration of the particles in the initial plasma. Such large values of the longitudinal momentum of electrons (positrons) in the laboratory frame are due to the transverse acceleration of electrons (positrons) in the moving frame. Performing the Lorentz transform back to the laboratory frame we obtain for the longitudinal momentum in the laboratory frame of the initial electrons and positrons $p_{\|} = \gamma_g[p_{\|0} + v_g(1 + p_{\|0}^2 + |\mathbf{a}^2|)^{1/2}] \approx \gamma_g v_g |\mathbf{a}|$ where we used $|\mathbf{a}| \gg |p_{\|0}|$.

In summary, the production of e^+e^- pairs by the electromagnetic wave propagation in the plasma leads to the up-shifting of the wave frequency and to the damping of the wave amplitude and changes the polarization state of the wave.

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