Pis'ma v ZhETF, vol. 80, iss. 11, pp. 839 – 842

Entanglement fidelity of coherent-state teleportation with asymmetric quantum channel

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Submitted 21 October 2004

A strategy for teleporting coherent states with the entanglement fidelity is considered in the general case of an asymmetric teleportation scheme. It is shown that the non-balanced homodyne detection with the subsequent coherent displacement is required to provide the average teleportation fidelity of entanglement.

PACS: 03.65.–w, 03.67.–a, 42.50.Dv

1. Introduction. Quantum teleportation is now considered as a key method for transmission of quantum states in communication protocols, among which quantum teleportation of continuous variables are of great importance [1 – 3]. An essential point of quantum teleportation lies in a quantum channel between the sending and receiving stations which is provided by a bipartite entangled state. In the case of continuous-variable quantum teleportation a two-mode squeezed vacuum state (TMSV) serves as the source of entanglement. The degree of entanglement is then controlled by the degree of squeezing, so that faithful teleportation requires the source of rather strong squeezing. Since the generation of highly squeezed states is a quite arduous problem, some improved protocols were proposed to maximize a quality of teleportation at finite squeezing [4 – 6]. In reality, however, because of unavoidable environment influences, TMSV turns into a mixed state that leads to further degradation of the initial entanglement and thus makes teleportation still worse. The consideration of continuous-variable quantum teleportation in noisy channels was a subject of a number of papers. In particular, schemes for increasing the degree of entanglement by purification were suggested [7, 8] and the influence of thermal bath on propagation of the entangled modes were analyzed [9 – 11].

The criterion that displays an efficiency of teleportation is its average fidelity. However, in general such a fidelity does not directly reveal the preserved amount of entanglement in the quantum channel and thus cannot serve as a sensitive measure of teleportation. For this, the *entanglement* fidelity is required [12]. The entanglement fidelity was shown to equal the standard fidelity of teleportation of pure states in the case of the symmetric Gaussian channel and was first obtained by Braunstein and Kimble [2]. Their teleportation protocol was based on the balanced homodyne detection in the sending station and the proper coherent displacement in the receiving station. As to teleportation with an asymmetric quantum channel, in the framework of the given protocol such an equivalence between the fidelities is no longer true, since teleportation becomes dependent on transmitting states [11].

This Letter is aimed to demonstrate the possibility to realize teleportation with the entanglement fidelity for asymmetric teleportation schemes. As the teleporting states, coherent states are assumed to be taken which play a significant role in optical communication. In this way, the non-balanced homodyne detection is to be applied in the sending station depending on the propagation paths of the modes from the source of entanglement, that, in its turn, completely determines the corresponding displacement operation in the receiving station.

2. The teleportation scheme. Let us consider the scheme of continuous-variable single-mode teleportation sketched in Fig.1. In this scheme TMSV is used as the source of entanglement. Assuming that the initially squeezed modes (with the squeezing parameter $\zeta = |\zeta| e^{i\varphi}$) propagate to the sending station (Alice) and to the receiving station (Bob) through the paths having the transmission coefficients T_1 and T_2 and embedded in the thermal baths at temperatures ϑ_1 and ϑ_2, respectively. Then the resulting state will be described by the following Wigner function [13]

$$W^{\mathrm{E}}(\alpha,\beta) = \frac{4}{\pi^2 \mathcal{N}} \times \\ \times \exp\left[-2\left(C_2|\alpha|^2 + C_1|\beta|^2 + S^*\alpha\beta + S\alpha^*\beta^*\right)\right], \quad (1)$$

where $(i = 1, 2)$

$$S = \frac{e^{i\varphi}}{\mathcal{N}} T_1 T_2 \sinh 2|\zeta|, \quad (2)$$

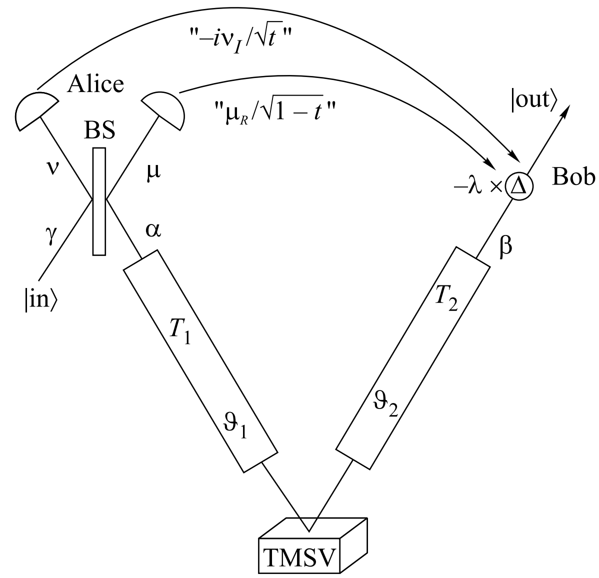

Fig.1. The teleportation scheme for transmitting quantum states from Alice to Bob. In quotes it is shown information that Alice sends to Bob using the classical channel after her non-balanced homodyne measurements by the beam splitter of transmissivity t

$$C_i = \frac{1}{\mathcal{N}}\left[1+|T_i|^2(\cosh 2|\zeta|-1)+2n_{\mathrm{th}\,i}\left(1-|T_i|^2\right)\right], \tag{3}$$

$$\mathcal{N}=\left[1+|T_1|^2(\cosh 2|\zeta|-1)+2n_{\mathrm{th}\,1}\left(1-|T_1|^2\right)\right]\times \times\left[1+|T_2|^2(\cosh 2|\zeta|-1)+2n_{\mathrm{th}\,2}\left(1-|T_2|^2\right)\right]- -|T_1T_2|^2\sinh^2 2|\zeta|, \tag{4}$$

with $n_{\mathrm{th}\,i}=\{\exp[\hbar\omega/(k_{\mathrm{B}}\vartheta_i)]-1\}^{-1}$ being the corresponding mean numbers of thermal excitations. The Gaussian form of the Wigner function (1) makes possible to estimate analytically the residual amount of entanglement, which can be still preserved in the (mixed) state shared by Alice and Bob. According to ref. [14], the measure of entanglement is determined by the expression

$$E=-\frac{1}{2}\log_2\left\{\frac{\mathcal{N}^2}{2}\left[C_1^2+C_2^2+2|S|^2- -(C_1+C_2)\sqrt{(C_1-C_2)^2+4|S|^2}\right]\right\}, \tag{5}$$

in case E takes on positive values, and ascribed to zero otherwise, i.e., when the state is appeared to become separable (see [15]). In the particular case of symmetric paths of the mode propagation ($C_1=C_2\equiv C$), the expression (5) is simplified to be

$$E=-\log_2\mathcal{N}(C-|S|). \tag{6}$$

In order to make use of the available quantum channel for teleporting an input quantum state, Alice performs a homodyne measurement on this mode and the mode being disposed at the sending station from the source by combining them through a (lossless) beam splitter. It is worth noting here that the beam splitter is presumed to be not obligatory the 50%:50% one, unlike it implies in the standard approach, but having some arbitrary transmissivity t. The similar scheme was used in ref. [6] for maximizing the teleportation fidelity. If $W_{\mathrm{in}}(\gamma)$ is the Wigner function of the input state, then as a result of such a combination one obtains the Wigner function of the (three-mode) overall system to be

$$W_{\mathrm{S}}(\mu,\nu,\beta)= =W_{\mathrm{in}}(\mu\sqrt{t}-\nu\sqrt{1-t})W^{\mathrm{E}}(\mu\sqrt{1-t}+\nu\sqrt{t},\beta). \tag{7}$$

Making the measurement of the real part of μ, μ_{R}, and the imaginary part of ν, ν_{I}, Bob's mode will be found in a quantum state, whose Wigner function is given by

$$W_{\mathrm{B}}(\beta|\mu_{\mathrm{R}},\nu_{\mathrm{I}})=\frac{4}{\pi^2\mathcal{N}\sqrt{t(1-t)}}\exp\left[-\frac{2}{C_2\mathcal{N}}|\beta|^2\right]\times \times\iint d\gamma_{\mathrm{R}}d\gamma_{\mathrm{I}}\,W_{\mathrm{in}}(\gamma_{\mathrm{R}}+i\gamma_{\mathrm{I}})\exp\left[-2C_2\left|\left(\frac{\mu_{\mathrm{R}}}{\sqrt{1-t}}-i\frac{\nu_{\mathrm{I}}}{\sqrt{t}}\right)+\frac{S^*}{C_2}\beta-\left(\gamma_{\mathrm{R}}\sqrt{\frac{t}{1-t}}+i\gamma_{\mathrm{I}}\sqrt{\frac{1-t}{t}}\right)\right|^2\right]. \tag{8}$$

Then, resting upon the pair of Alice's measurement data μ_{R} and ν_{I} at the fixed transmissivity t of the beam splitter that enter eq. (8) in the form of

$$\Delta(\mu_{\mathrm{R}},\nu_{\mathrm{I}})\equiv\mu_{\mathrm{R}}/\sqrt{1-t}-i\nu_{\mathrm{I}}/\sqrt{t}, \tag{9}$$

Bob may coherently displace the quantum state of his mode by applying the displacement $\beta\to\beta-\lambda\Delta(\mu_{\mathrm{R}},\nu_{\mathrm{I}})$ with some arbitrary strength λ. Thus, after averaging over all the measurement outcomes, it will create the teleported quantum state on average:

$$W_{\mathrm{out}}(\beta)=\iint d\mu_{\mathrm{R}}d\nu_{\mathrm{I}}\,W_{\mathrm{B}}(\beta-\lambda\Delta(\mu_{\mathrm{R}},\nu_{\mathrm{I}})|\mu_{\mathrm{R}},\nu_{\mathrm{I}}). \tag{10}$$

As a criterion of teleportation one can use the fidelity of the final state (10) to the initial one, which in the case of a pure input state is given by the overlap of their Wigner functions

$$F=\pi\iint d^2\beta\,W_{\mathrm{in}}(\beta)W_{\mathrm{out}}(\beta). \tag{11}$$

This fidelity is evidently dependent on the teleportation protocol as a whole and so can only optionally display the character of the quantum channel. Taking as an example of teleportation of coherent states, it will be shown how the fidelity (11) can be turned into the fidelity of entanglement for the general case of the asymmetric quantum channel.

3. Teleportation of coherent states. Now apply the teleportation scheme from the preceding section for transmitting coherent states. Performing all steps of the teleportation protocol, such as homodyne detection with the beam splitter of transmissivity t and the coherent displacement of the strength λ, after straightforward calculations one can arrive at the following expression for the fidelity of teleportation of the coherent state $|\alpha_0\rangle$

$$F_{\mathrm{coh}}(\alpha_0) =$$

$$= \frac{2\exp\left\{-\dfrac{2(\sqrt{1-t}-\lambda\sqrt{t})^2}{1-t+\lambda^2 t+(1-t)\sigma(\lambda)}\,\alpha_0^2\right\}}{\sqrt{\left[\sigma(\lambda)+\lambda^2\dfrac{t}{1-t}+1\right]\left[\sigma(\lambda)+\lambda^2\dfrac{1-t}{t}+1\right]}}, \qquad (12)$$

where $\sigma(\lambda) = \mathcal{N}(C_2 + \lambda^2 C_1 - 2\lambda|S|)$. This fidelity is a generalization of the analogous expression obtained in ref. [11] for the case of balanced homodyning, i.e., when $t = 0.5$. At large α_0 the fidelity (12) is seen to have the exponentially suppression factor unless

$$\lambda = \sqrt{\frac{1}{t} - 1}, \qquad (13)$$

at which the dependence on the coherent amplitude entirely disappears, so that the fidelity looks as

$$F_{\mathrm{coh}} = \frac{2}{\sqrt{[\sigma(\lambda)+2][\sigma(\lambda)+\lambda^4+1]}}. \qquad (14)$$

For the symmetric quantum channel, when balanced homodyning, $t = 0.5$, is to be used, the choice of the strength of the coherent displacement $\lambda = 1$ satisfies the suppression condition (13) and leads the fidelity (14) to the form

$$F_{\mathrm{coh}}^{\mathrm{sym}} = \frac{2}{2+\sigma^{\mathrm{sym}}(1)} = \frac{1}{1+\mathcal{N}(C-|S|)}. \qquad (15)$$

Taking into account the relation (6), one can see that the fidelity (15) is nothing but the entanglement fidelity of teleportation

$$F_{\mathrm{coh}}^{\mathrm{E}} = \frac{1}{1+2^{-E}}. \qquad (16)$$

For the entangled teleportation channel, $E > 0$, this fidelity lies above 0.5, approaching unity for the infinite entanglement (the case of perfect teleportation). Thus, the value 0.5, corresponding to $E = 0$, can be considered as the classical limit of teleportation, below which entanglement in the quantum channel is absent.

However, to assure the entanglement fidelity (16) in an asymmetric case, balanced homodyning turns out to be unacceptable. Indeed, even if λ, according to eq. (13), is chosen to equal 1 to eliminate the suppression factor, anyway the fidelity (12) can be below the classical limit 0.5 at the entangled teleportation channel. It is due to the asymptotic behaviour of σ at the large squeezing parameter $|\zeta|$

$$\sigma(\lambda) \simeq \frac{1}{2}\left(|T_2| - \lambda|T_1|\right)^2 e^{|2\zeta|} \qquad (17)$$

that disperses at $\lambda = 1$ for $|T_1| \neq |T_2|$. Thus, the fidelity of teleportation in such a case

$$F_{\mathrm{coh}} = \frac{2}{2+\sigma(1)} \simeq \frac{1}{1+\frac{1}{4}\left(|T_2|-|T_1|\right)^2 e^{|2\zeta|}}$$

diminishes to zero at the large initial squeezing regardless of the entanglement increase.

The way out to obtain the entanglement fidelity in an asymmetric teleportation scheme can be found by means of non-balanced homodyning. The transmissivity t of the beam splitter is then no longer fixed to 0.5, but must be determined with the help of the relation (13) by the adjustment of the coherent displacement strength λ to fulfil the equivalence between the fidelities (14) and (16). The result of such fitting will be varied depending on the specific conditions of the mode propagation and source squeezing. The choice of t will also fix the base Δ of the coherent displacement in eq. (9), so that all steps of the teleportation strategy to provide the fidelity of entanglement become completely definite.

An example demonstrating how this procedure can be done for the particular case of the mode propagation is illustrated with Figs.2 and 3. The behaviour of the entanglement fidelity in dependence on the source squeezing is shown in Fig.2. The fidelity is seen to exceed the classical limit 0.5 starting only from $|\zeta| \approx 0.75$ that indicates the absence of entanglement between the modes in Alice's and Bob's hands at the squeezing parameter of the source below this value. It means that for $|\zeta| < 0.75$ the corresponding initial entanglement between the modes, provoked by the squeezing, is entirely wasted because of absorption losses in their propagation from the source. The proper choice of the strength λ of the coherent displacement and the transmissivity t as the result of the computer adjustment to provide this entanglement fidelity of teleportation are plotted in Fig.3. It can be seen that at large $|\zeta|$ the strength λ tends to

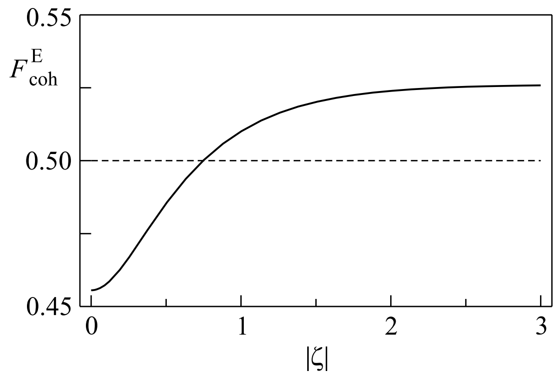

Fig.2. The entanglement fidelity (16) of coherent-state teleportation is shown as a function of $|\zeta|$ at $|T_1| = 0.95$, $|T_2| = 0.85$ and $n_{\mathrm{th}\,1} = 1$, $n_{\mathrm{th}\,2} = 2$ in accordance with the definition (5)

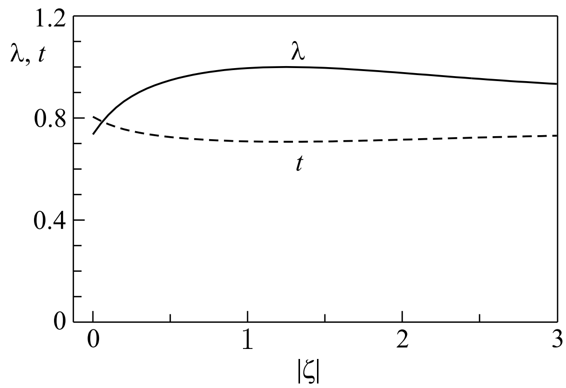

Fig.3. The strength λ of the coherent displacement (solid line) and, according to eq. (13), the transmissivity t of the beam splitter (dotted line), which provide coherent-state teleportation with the entanglement fidelity, are shown in dependence on $|\zeta|$ at the same propagation parameters as in Fig.2

$|T_2|/|T_1| \approx 0.9$, which protects σ from dispersion in accordance with eq. (17).

4. Conclusion. A teleportation scheme with the mixed quantum channel has been analysed from the point of view of realizing the transmission of coherent states with the entanglement fidelity. The mixed teleportation channel is associated with absorption losses in the transmission of the TMSV modes along the paths to the sending and receiving stations.

It has been shown that the entanglement fidelity of teleportation with the asymmetric quantum channel could be achieved by means of Bob's specific coherent displacement resting on results of Alice's non-balanced homodyne measurement. However, in order to match the necessary transmissivity of the measurement beam splitter and the corresponding parameters of the displacement, one needs to know information on the initial squeezing in the source as well as the conditions of the mode propagation. This fact fundamentally differs the case of the asymmetric quantum channel from the symmetric one, when teleportation with the entanglement fidelity occurs at the fixed parameters being independent on the scheme.

This work was supported by the Heisenberg-Landau Program. I would like to acknowledge helpful discussions with L. Knöll and D.-G. Welsch.

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