Pis'ma v ZhETF, vol. 80, iss. 2, pp. 81 – 85 © 2004 July 25

Vector meson couplings to vector and tensor currents in extended NJL quark model

M. Chizhov

Institut de Physique Nucléaire de Lyon and Université Claude Bernard Lyon-1, 69622 Villeurbanne, France

Centre for Space Research and Technologies, Faculty of Physics, University of Sofia, 1164 Sofia, Bulgaria

Submitted 14 May 2004

A simple explanation of the dynamic properties of vector mesons is given in the framework of extended Nambu–Jona-Lasinio quark model. New mass relations among the hadron vector resonances are derived. The results of this approach are in good accordance with the QCD sum rules, the lattice calculations and the experimental data.

PACS: 12.39.Ki, 14.40.Cs

I. Introduction. All known elementary vector particles, the photon, Z,W and the gluons, are gauge particles. An opinion exists that there are no other vector particles besides the gauge ones. They have only chirality conserved vector interactions with the matter fields. The so called *anomalous* term, which appears from radiative corrections [1], is small and is not present in the initial Lagrangian. It describes chirality flipped interactions with the tensor current.

Although the gluons are gauge particles, they induce chirality violation and, therefore, in the hadron physics the *anomalous* interactions of the vector mesons are not so small. They should be taken into account, for example to extract $|V_{ub}|$ element of CKM matrix from the semileptonic decay $B \to \rho\ell\nu$. Even more peculiar is the existence of the vector $b_1(1235)$ meson with only anomalous interactions with quarks. It is not possible at all to describe this meson as a gauge particle.

Such kind of particles originate from a tensor formalism. It is known that free relativistic particles with spin 1 can be described by the four-vector A_μ or by the second rank antisymmetric tensor field $T_{\mu\nu}$. These two different descriptions are applied to the vector mesons without noting the main difference between them [2]. However, the different formalisms are related to the different couplings of the vector mesons to the quarks and, therefore, to the different chiral properties. The vector fields are transformed under *real* (chirally neutral) representation (1/2,1/2) of the Lorentz group. On the other hand $T_{\mu\nu} \pm i\tilde{T}_{\mu\nu}$ combinations are transformed under irreducible *chiral* representations (1,0) and (0,1). They describe *chiral* vector fields.

The aim of this letter is to put on the same footing (at least at a phenomenological level) the consideration of the gauge particles and the particles with *anomalous* only interactions. The successful description of the dynamical properties of the hadron systems hints that the same phenomenon takes place in the high energy physics too. Then we may expect the discovery of *anomalously* interacting particles at future colliders.

The Nambu–Jona-Lasinio (NJL) [3] quark model is a successful tool for investigating hadron physics and the spontaneous chiral symmetry breaking mechanism. However, an extension of the model is needed [4] in order to introduce the new type *anomalous* interactions and the particles associated with them .

To simplify the idea we will deal only with the one-flavor NJL model. The generalization for the real case with N flavors is straightforward and will be considered elsewhere. As long as the isospin triplet $I = 1$ consists of *up* and *down* quarks with approximately the same constituent masses, we can apply this one-flavor model to the vector mesons ρ, b_1, a_1 and ρ'. Using this approach we will get, for example, very simple prediction for the ratio $f^T_\rho/f_\rho \simeq 1/\sqrt{2}$, which matches very well the latest lattice calculations [5].

II. The model. Following the classical paper [3] we start with the chiral invariant Lagrangian

$$\mathcal{L} = \bar\psi i\partial\!\!\!/\psi + \frac{G_0}{2}\,\bar\psi(1+\gamma^5)\psi\;\bar\psi(1-\gamma^5)\psi - \\ - \frac{G_V}{2}\left(\bar\psi\gamma_\mu\psi\right)^2 - \frac{G_A}{2}\left(\bar\psi\gamma_\mu\gamma^5\psi\right)^2 - \\ - \frac{G_T}{2}\,\bar\psi\sigma_{\mu\lambda}(1+\gamma^5)\psi\frac{q_\mu q_\nu}{q^2}\bar\psi\sigma_{\nu\lambda}(1-\gamma^5)\psi, \qquad (1)$$

where the new tensor interaction term was introduced. It should contain an unique momentum dependence, because the local product of two tensor currents with different chiralities vanishes identically. Generally speaking, there are four different positive coupling parameters

G_0, G_V, G_A and G_T with dimensions $[mass]^{-2}$ for each chiral invariant term.

Let us rewrite the Lagrangian (1) by introducing auxiliary bosonic fields (without kinetic terms) which will later play the role of collective meson states after quantization

$$\mathcal{L}=\bar\psi\not{\partial}\psi + g_\sigma\bar\psi\psi\sigma - \frac{g_\sigma^2}{2G_0}\sigma^2 + ig_\pi\bar\psi\gamma^5\psi\pi - \frac{g_\pi^2}{2G_0}\pi^2 + \\ + g_V\bar\psi\gamma_\mu\psi V_\mu + \frac{g_V^2}{2G_V}V_\mu^2 + g_A\bar\psi\gamma_\mu\gamma^5\psi A_\mu + \frac{g_A^2}{2G_A}A_\mu^2 - \\ - ig_R\,\bar\psi\sigma_{\mu\nu}\psi\,\frac{q_\mu}{|q|}R_\nu + \frac{g_R^2}{2G_T}R_\mu^2 + \\ + g_B\,\bar\psi\sigma_{\mu\nu}\gamma^5\psi\,\frac{q_\mu}{|q|}B_\nu + \frac{g_B^2}{2G_T}B_\mu^2, \qquad (2)$$

where g_a $(a = \sigma, \pi, V, A, R, B)$ are dimensionless coupling constants. At the classical level the Lagrangians (1) and (2) are equivalent. However, the perturbation quantum field theory cannot be applied in the former case due to the dimensional constants G. Therefore, the second form of the Lagrangian with dimensionless coupling constants g_a is more appropriate for quantization by perturbative methods [6].

III. Quantum corrections and symmetry breaking. It is interesting to note that in spite of absence of kinetic terms for the meson fields in the initial Lagrangian (2) they are generated on the quantization stage due to radiative corrections (Fig.1a). Such kinetic

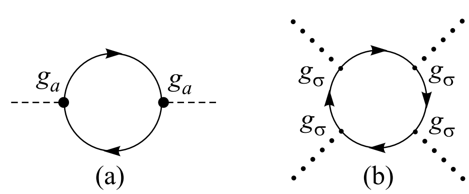

Fig.1. Radiative corrections to (a) meson self-energy and (b) scalar self-interaction

terms generation reflects into relations of the coupling constants g_a. In one-loop approximation they are

$$3g_\sigma^2 = 3g_\pi^2 = 2g_V^2 = 2g_A^2 = g_R^2 = g_B^2 = \frac{24\pi^2}{N_c}\,\varepsilon, \qquad (3)$$

where N_c is the number of colors and ε is the dimensional regularization parameter. The quark loops lead also to various self-interactions and interactions among the mesons. For example, due to generation of quartic self-interaction σ^4 of the scalar meson (Fig.1b), a spontaneous breaking of the chiral symmetry occurs which leads to generation of non-zero quark mass $m = -\langle\sigma\rangle/g_\sigma$, to additional mass contributions for the

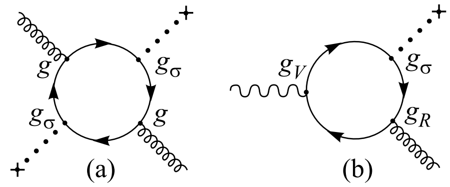

Fig.2. Contributions to the mass term (a) and mixing between vector mesons (b)

mesons (Fig.2a) and to mixings between them (Fig.2b).

The chiral symmetry breaking for the scalar sector has been studied in detail. The introduction of the new vector mesons R_μ and B_μ brings nothing new there. On the other hand the symmetry breaking leads to an interesting phenomenon in the vector mesons sector. In the following we focus just on the vector mesons.

We have introduced four vector mesons V_μ, A_μ, R_μ and B_μ. Their quantum numbers (the total angular momentum J, P-parity and the charge conjugation C) can be defined from their interactions with quarks (2):

meson vector fields:	V_μ	A_μ	R_μ	B_μ
quantum numbers J^{PC}:	1^{--}	1^{++}	1^{--}	1^{+-}

The vector mesons V_μ and A_μ have gauge-like minimal interactions with quarks, while R_μ and B_μ have only *anomalous* tensor interactions. Nevertheless, in the chiral limit all these mesons couple to *conserved* quark currents. Therefore, their kinetic terms are gauge invariant

$$\mathcal{L}_0 = -\frac{1}{2}V_\mu\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)V_\nu + \frac{M_V^2}{2}V_\mu^2 - \\ - \frac{\sqrt{18}m}{|q|}\,V_\mu\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)R_\nu - \\ - \frac{1}{2}R_\mu\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)R_\nu + \frac{M_T^2 - 6m^2}{2}R_\mu^2 - \\ - \frac{1}{2}B_\mu\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)B_\nu + \frac{M_T^2 + 6m^2}{2}B_\mu^2 - \\ - \frac{1}{2}A_\mu\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)A_\nu + \frac{M_A^2 + 6m^2}{2}A_\mu^2. \qquad (4)$$

Here $M_V^2 = g_V^2/G_V$, $M_A^2 = g_A^2/G_A$ and $M_T^2 = g_R^2/G_T = g_B^2/G_T$ are the initial vector boson masses before symmetry breaking. Since R_μ and B_μ are chiral mesons, their masses $M_R^2 = M_T^2 - 6m^2$ and $M_B^2 = M_T^2 + 6m^2$ split due to symmetry breaking. The axial-vector mesons A_μ also get additional contribution to their mass terms.

The essential feature of eq. (4) is the mixing between the two vector mesons V_μ and R_μ which have the same quantum numbers as ρ and ρ' mesons. In the Lorentz gauge this reads

$$\mathcal{L}_{VR} = -\frac{1}{2}\left(V_\mu R_\mu\right)\begin{pmatrix} q^2 - M_V^2 & \sqrt{18}m|q| \\ \sqrt{18}m|q| & q^2 - M_R^2 \end{pmatrix}\begin{pmatrix} V_\mu \\ R_\mu \end{pmatrix}. \quad (5)$$

If we identify the physical states with quantum numbers 1^{--} with these mesons, they can be expressed as linear combinations of the chiral eigenstates V_μ and R_μ:

$$\begin{aligned} \rho_\mu(q^2) &= \quad \cos\theta(q^2)V_\mu + \sin\theta(q^2)R_\mu, \\ \rho'_\mu(q^2) &= -\sin\theta(q^2)V_\mu + \cos\theta(q^2)R_\mu, \end{aligned} \quad (6)$$

where

$$\tan 2\theta(q^2) = \frac{\sqrt{72m^2q^2}}{M_R^2 - M_V^2}. \quad (7)$$

It means that ρ and ρ' mesons have both vector and anomalous tensor couplings with quarks, while the axial-vector mesons a_1 with quantum numbers 1^{++}, which are assigned to A_μ, have only gauge-like minimal interactions and the axial-vector mesons b_1 with quantum numbers 1^{+-}, which are assigned to B_μ, have only anomalous tensor interactions.

Diagonalization of (5) leads to relations for the physical masses of ρ, ρ' and b_1 mesons [7]: $m_\rho = 771.1 \pm 0.9$ MeV, $m_{\rho'} = 1465 \pm 25$ MeV and $m_{b_1} = 1229.5 \pm 3.2$ MeV:

$$\begin{aligned} m_\rho^2 + m_{\rho'}^2 &= M_V^2 + m_{b_1}^2 + 6m^2, \\ m_\rho^2 m_{\rho'}^2 &= M_V^2\left(m_{b_1}^2 - 12m^2\right), \end{aligned} \quad (8)$$

which define $M_V = 1034 \pm 33$ MeV and the quark mass $m = 163 \pm 7$ MeV.

In general, the mixing angle θ depends on q^2 (7) and should be different at ρ- and ρ'-scale. However, the denominator in (7) is smaller in comparison with the nominator and $\tan 2\theta(q^2) = |q|/(89^{+82}_{-75}$ MeV) is big, which corresponds to almost maximal mixing $\theta \sim \pi/4$ with weak q^2-dependence.

If we suppose, that the effective four-fermion interactions of quarks (1) could originate in QCD by gluon exchange in $1/N_c$ limit, it follows $M_V = M_A$ [8]. Then from the first equation in (8) a remarkable relation among masses of the vector mesons is obtained

$$m_\rho^2 + m_{\rho'}^2 = m_{a_1}^2 + m_{b_1}^2. \quad (9)$$

This leads to a little bit smaller value for the mass of a_1 meson $m_{a_1} = 1109 \pm 37$ MeV (2.2σ below PDG estimation).

IV. Longitudinal and transverse polarizations of the vector mesons. The longitudinal and transverse polarizations of the (axial-)vector mesons are defined by corresponding couplings f from the matrix elements

$$\begin{aligned} \langle 0|\bar\psi\gamma_\mu\gamma^5\psi|A(q,\lambda)\rangle &= m_A f_A e_\mu^\lambda, \\ \langle 0|\bar\psi\sigma_{\mu\nu}\psi|A(q,\lambda)\rangle &= i f_A^T \varepsilon_{\mu\nu\alpha\beta} e_\alpha^\lambda q_\beta; \end{aligned} \quad (10)$$

$$\begin{aligned} \langle 0|\bar\psi\gamma_\mu\psi|V(q,\lambda)\rangle &= m_V f_V e_\mu^\lambda, \\ \langle 0|\bar\psi\sigma_{\mu\nu}\psi|V(q,\lambda)\rangle &= i f_V^T\left(e_\mu^\lambda q_\nu - e_\nu^\lambda q_\mu\right), \end{aligned} \quad (11)$$

where e_μ^λ is the polarization vector of a spin-1 meson. Then, using the meson-fermion couplings (2) and taking into account the mixing after the symmetry breaking (6), we can express the couplings f in terms of the parameters of our model:

$$f_{a_1} = m_{a_1}/g_A, \quad f_{a_1}^T = 0; \quad (12)$$

$$f_{b_1} = 0, \quad f_{b_1}^T = m_{b_1}/g_B; \quad (13)$$

$$\begin{aligned} f_\rho &= \frac{m_\rho\cos\theta(m_\rho) + \sqrt{18}m\sin\theta(m_\rho)}{g_V}, \\ f_\rho^T &= \frac{m_\rho\sin\theta(m_\rho) + \sqrt{18}m\cos\theta(m_\rho)}{g_R}; \end{aligned} \quad (14)$$

$$\begin{aligned} f_{\rho'} &= \frac{-m_{\rho'}\sin\theta(m_{\rho'}) + \sqrt{18}m\cos\theta(m_{\rho'})}{g_V}, \\ f_{\rho'}^T &= \frac{m_{\rho'}\cos\theta(m_{\rho'}) - \sqrt{18}m\sin\theta(m_{\rho'})}{g_R}; \end{aligned} \quad (15)$$

where g_a obey the relations (3).

We can make immediately simple qualitative predictions, using the interesting fact that the solution of the system (8) is very close to the unique solution when the mixing angle θ equals $\pi/4$ and does not depend on q^2. This happens when $M_R^2 = M_V^2$. In this case some valuable mass relations, besides (9), have place

$$\begin{aligned} m_{\rho'} &= m_\rho + \sqrt{18}m, \\ 3m_{b_1}^2 &= 2m_{\rho'}^2 - m_\rho m_{\rho'} + 2m_\rho^2, \\ 3m_{a_1}^2 &= m_{\rho'}^2 + m_\rho m_{\rho'} + m_\rho^2. \end{aligned} \quad (16)$$

The first two relations based only on the suggestion of maximal mixing, while the last one requires of the additional constraint $M_V = M_A$. Using more precise mass values for ρ and b_1 mesons [7], we can predict other masses based solely on this ansatz: $m_{a_1} = 1155.1 \pm 2.7$ MeV, $m_{\rho'} = 1500.5 \pm 4.8$ MeV and $m = 171.9 \pm 1.3$ MeV.

It is interesting to note that the second relation can be applied to the spin-1 isosinglets $I = 0$ $\omega(782)$, $\omega'(1420)$, $h_1(1170)$ and $\phi(1020)$, $\phi'(1680)$, $h_1(1380)$ which are almost pure $(u\bar u + d\bar d)/\sqrt{2}$ and $s\bar s$ states correspondingly. In the first case the mass relation among

ω's and h_1 is fair satisfied. In the second case we can confirm existence of $h_1(1380)$ state which is omitted from summary table. Our prediction for the mass $m_{h_1(1380)} = 1415 \pm 13$ MeV agrees with PDG average $m_{h_1(1380)} = 1386 \pm 19$ MeV.

In the case of the maximal mixing the eqs. (14) and (15) can be rewritten in a compact form

$$f_\rho = \frac{m_{\rho'}}{\sqrt{2} g_V}, \qquad f_\rho^T = \frac{m_{\rho'}}{\sqrt{2} g_R}; \tag{17}$$

$$f_{\rho'} = -\frac{m_\rho}{\sqrt{2} g_V}, \qquad f_{\rho'}^T = \frac{m_\rho}{\sqrt{2} g_R}. \tag{18}$$

As far as $g_R = \sqrt{2} g_V$ Eq. (17) leads directly to a prediction for the ratio $f_\rho^T / f_\rho = 1/\sqrt{2} \approx 0.707$, which is in perfect agreement with the latest lattice calculations [5]. The analogous ratio for the ρ' meson should have the same value and an opposite sign $f_{\rho'}^T / f_{\rho'} = -1/\sqrt{2}$. Unfortunately, there are no lattice calculations for the ρ' meson yet.

The another reliable method, giving the information about the matrix elements, is the QCD sum rules [9]. If one accepts the experimental value for the vector coupling $f_\rho = 208 \pm 10$ MeV [5] the QCD sum rules give compatible with lattice calculation result $f_\rho^T = 160 \pm 10$ MeV [10]. It is interesting to note that the consideration of the correlation function of the tensor with the vector current proves that the relative sign of f_ρ^T and f_ρ is positive [11]. Moreover, it was shown [12] that the contribution of ρ' meson to this correlation function is negative and $m_\rho f_\rho f_\rho^T \simeq -2 m_{\rho'} f_{\rho'} f_{\rho'}^T$. That is in a good accordance with our predictions (17), (18), since $m_{\rho'} \simeq 2 m_\rho$.

The derivation of the transverse couplings within the QCD sum rules framework has been re-estimated in [13]: $f_\rho^T = 157 \pm 5$ MeV and $f_{b_1}^T = 184 \pm 5$ MeV, confirming the results of [10]. In addition the ρ'-transverse coupling $f_{\rho'}^T = 140 \pm 5$ MeV has been evaluated, which is, however, in contradiction with the superconvergence relation

$$\left(f_\rho^T\right)^2 + \left(f_{\rho'}^T\right)^2 = \left(f_{b_1}^T\right)^2 \tag{19}$$

from [14].

It should be compared with our predictions for the same couplings: $f_\rho^T = (0.703^{+0.004}_{-0.007}) f_\rho = 146 \pm 7$ MeV, $f_{b_1}^T = (0.839^{+0.017}_{-0.015}) f_\rho = 175 \pm 9$ MeV and $f_{\rho'}^T = (0.405^{+0.040}_{-0.034}) f_\rho = 84 \pm 9$ MeV. The first two of them are in good agreement with all QCD sum rules results, while the last one for the ρ' meson is in a good accordance with the relation (19), but it is in noticeable disagreement with [13].

It is worth noticing that the anomalous dimension of the tensor current is not zero and the couplings $f^T(\mu)$ are scale dependent. Our values are systematically lower than QCD estimations made at the renormalization scale $\mu = 1$ GeV and closer to lattice calculations at $\mu = 2$ GeV.

V. Conclusion. In this letter we followed an approach based on the extended NJL quark model, describing all low-lying meson resonances. Totally there are 16 degrees of freedom for the one-flavor quark-antiquark meson excitations: scalar, pseudoscalar, vector, axial-vector and antisymmetric tensor. They have corresponding Yukawa interactions with quarks. All these excitations are assigned to physical meson states σ, π, ρ, a_1, ρ' and b_1. The (pseudo)scalar sector was already well studied and introduction of the tensor bosons does not bring anything new there. Hence, we have concentrated on the spin-1 meson sector mainly.

Simultaneous description of all these states in the framework of NJL model leads to interesting predictions like new mass formulas and relations among meson coupling constants. All these relations are in agreement with present experimental data and the numerical calculations on the lattice and the QCD sum rules.

Other interesting property which follows immediately from our approach is the dual nature of ρ and ρ' mesons. They have both vector and tensor couplings with quarks. The new insight on this phenomenon is the suggestion that there exist two different vector particles with the same quantum numbers, which interact differently with quarks. One of them has only gauge-like vector interactions and the other one has only *anomalous* tensor interactions. After the spontaneous chiral symmetry breaking they are mixed, producing the physical ρ and ρ' meson states. From the hadron phenomenology point of view this suggestion does not seem unnatural, because the axial-vector mesons a_1 and b_1, due to their different quantum numbers, exist as pure states, which have correspondingly only gauge-like couplings and tensor couplings with quarks.

The above consideration means that in Nature there exist two different vector particles with respect to their interactions with matter. However, they are just composite quark-antiquark states. The question, if there are new fundamental vector particles, will be probably answered at the future colliders and especially at LHC.

I would like to thank Professor V. Braun for useful comments. During preparation of this letter I learned about premature death of Professor Ian Kogan. All we miss him very much. I acknowledge the warm hospitality of IPNL and especially Professors S. Katsanevas and Y. Déclais.

1. J. Schwinger, Phys. Rev. **73**, 416 (1948).
2. G. Ecker et al., Nucl. Phys. B**321**, 311 (1989).
3. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
4. M. V. Chizhov, hep-ph/9610220.
5. D. Becirevic et al., JHEP **05**, 007 (2003); V. M. Braun et al., Phys. Rev. **D68**, 054501 (2003).
6. T. Eguchi, Phys. Rev. **D14**, 2755 (1976).
7. K. Hagiwara et al., Phys. Rev. **D66**, 010001 (2002).
8. J. Bijnens, Phys. Rep. **265**, 369 (1996).
9. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
10. P. Ball and V. M. Braun, Phys. Rev. **D54**, 2182 (1996).
11. V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
12. V. M. Belyaev and Ya. I. Kogan, Sov. J. Nucl. Phys. **40**, 659 (1984).
13. A. P. Bakulev and S. V. Mikhailov, Eur. Phys. J. C**17** 129 (2000).
14. N. S. Craigie and J. Stern, Phys. Rev. **D26**, 2430 (1982).