Flow phase diagram for the helium superfluids

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The existence of the flow phase diagram predicted by Volovik [JETP Letters **78**, 553 (2003)] is discussed based on available experimental data for He II and ^{3}He-B. The effective temperature-dependent but scale-independent Reynolds number $\mathrm{Re}_{\mathrm{eff}} = 1/q \equiv (1-\alpha')/\alpha$, where α and α' are the mutual friction parameters and the superfluid Reynolds number characterizing the circulation of the superfluid component in units of the circulation quantum are used as the dynamic parameters. In particular, the flow diagram allows identification of experimentally observed turbulent states I and II in counterflowing He II with the classical and quantum turbulent regimes suggested by Volovik.

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We consider the flow of quantum liquids such as He II or ^{3}He-B that can be described in the framework of the two fluid model (see, e.g., [1]). The normal fluid and superfluid velocity fields are coupled by two terms: the Gorter–Mellink term that describes the mutual friction between these two liquids when vortices are present in the superfluid, and by the temperature gradient term, responsible, e.g., for the fountain effect. Circulation in the superfluid component is quantized in units of κ ($0.997 \cdot 10^{-3}$ cm^2/s for He II and $0.662 \cdot 10^{-3}$ cm^2/s for ^{3}He-B); we assume singly quantized vortices.

Let us consider a flow that can be approximated as isothermal[2)]. Then the generally coupled complex flow of both components described by the two two-fluid equations can be simplified and becomes easier to understand, especially for two extreme cases:

i) *There are no quantized vortices in the flow.* This represents a situation when the normal and superfluid velocity fields are fully decoupled. The normal fluid thus obeys the usual Navier-Stokes equation while the superfluid flow remains potential. Thus formally the normal fluid could become turbulent without a single vortex being present in the superfluid - in the absence of mutual friction the superfluid simply does not "know" what is happening in the normal fluid. In practice, however, remnant vortices are almost always present, at least in He II [2], pinned to walls which are always rough on the atomic scale. In ^{3}He-B a vortex free sample is more likely, but the highly viscous normal fluid can hardly become turbulent in a laboratory sized container.

ii) *The normal fluid is at rest in some frame of reference.* Such a possibility arises, for example, for ^{3}He-B, whose highly viscous normal component is effectively clamped by the walls in a laboratory size container[3)]. Following first the original approach of Volovik [3, 4], let assume that the quantized vortices in the flow are arranged in such a way that the coarse-grained hydrodynamic equation

$$\frac{\partial \mathbf{v}_s}{\partial t} + \nabla\mu = (1-\alpha')\mathbf{v}_s \times \boldsymbol{\omega} + \alpha\hat{\boldsymbol{\omega}} \times (\boldsymbol{\omega} \times \mathbf{v}_s), \quad (1)$$

obtained from the Euler equation after averaging over vortex lines [5], written in the frame of reference of the normal fluid, provides a sufficiently accurate description of the superflow. We shall return to the applicability of this equation later. The normal fluid thus provides a unique frame of reference and we have to deal only with the superfluid velocity $\mathbf{v}_s$. Re-scaling of the time variable such that $t \to 1-\alpha' t$ leads to

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla\mu = \mathbf{v} \times \boldsymbol{\omega} + q\hat{\boldsymbol{\omega}} \times (\boldsymbol{\omega} \times \mathbf{v}), \quad (2)$$

where $\mathbf{v} = \mathbf{v}_s$, $\boldsymbol{\omega}$ is the course-grained vorticity and $\hat{\boldsymbol{\omega}}$ is a unit vector in the direction of $\boldsymbol{\omega}$. The theoretical analysis of the fluid dynamical problem based on this equation has been performed by Volovik [3, 4], Vinen [6] and L'vov, Nazarenko and Volovik [7]. As was first emphasized by Finne et al. [8], Eq. (2) has a very remarkable property which makes it distinct from the ordinary Navier-Stokes equation where the relative importance of the inertial and dissipative terms is given by the Reynolds number, which in turn depends on the

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2) This cannot be strictly true, as dissipation in flowing normal fluid possessing finite viscosity leads to heating in places of high vorticity and to a counterflow. We assume here that the heating is small and can be neglected.

3) This situation can also occur in He II at fairly low temperature ($\simeq 1$ K and below), where kinematic viscosity of the normal fluid rapidly increases with decreasing temperature [9].

geometry of the particular flow under study. Here the role of the effective Reynolds number is played by the parameter $\mathrm{Re}_{\mathrm{eff}} = q^{-1} = (1-\alpha')/\alpha$ that depends on temperature but not on geometry. We stress that the superfluid Reynolds number (as soon as it is high enough, so that Eq. (2) represents a good approximation of the superflow) is not relevant to consideration of the problem of flow obeying Eq. (2), the beauty of which consists in the fact that one is able to derive more general conclusions about turbulent flow generated from suitable initial conditions depending only on a single temperature dependent parameter $1/q$, regardless of the actual geometry of the flow. A wide range of q values is easily experimentally achievable; with q increasing with temperature in both He II [9] and ^{3}He-B [10].

Like the usual Navier-Stokes equation, Eq. (2) has both laminar (for $q \gg 1$) and turbulent solutions. For $q \ll 1$ it describes fully developed turbulence. The latter is discussed in detail in [3, 4], showing that a turbulent cascade will develop, covering a range of scales. It was claimed, however, that the 3D energy spectrum is of usual[4)] Kolmogorov form $E(k) \cong \varepsilon^{2/3}k^{-5/3}$.

Vinen [6] recently developed a different approach to superfluid turbulence in the presence of a stationary normal fluid. His approach is based on physical arguments concerning the turnover and decay times of eddies of various sizes, and the results are confirmed by numerical solutions of a diffusion equation that describes flow of turbulent energy in k-space. Owing to the action of mutual friction, there is strong damping of large eddies, with the result that at low wave numbers the energy spectrum falls off much more rapidly (approaching k^{-3}) than for the Kolmogorov spectrum. However, the damping remains weak for small eddies, so that the Kolmogorov spectrum is recovered for large k, beyond a certain critical wave number. Vinen also correctly points out that this feature is inherently contained in Eq. (2).

The most recent theoretical discussion of this issue by L'vov, Nazarenko and Volovik [7], based on analytical solution of the first order k-space diffusion equation confirms the crucial role of mutual friction force on large scale. Moreover, scenarios of various turbulent regimes are suggested and discussed, depending on various parameters of the flow.

Although this interesting problem of superfluid turbulence in the presence of a stationary normal fluid is most likely not yet fully settled, we believe that the main features of such a turbulent superflow have been firmly established.

The continuous approach for considering superfluid turbulence based on Eq. (2) would be fully applicable in the limit $\kappa \to 0$. As pointed out by Volovik [3, 4], at finite κ one has to ensure that, at the smallest scale r_0, the "granularity" due to individual vortices does not become important, so that the circulation $v_{r0}r_0 = q^2UR = q^2\kappa\mathrm{Re}_s > \kappa$. This leads to an important criterion $\mathrm{Re}_s > 1/q^2 \gg 1$. Indeed, the turbulent cascade might reach small scales containing only a few quantized vortices and most likely continues (perhaps in a form of a Kelvin wave cascade), but the form of the energy spectrum around and beyond the quantum scale [11], $\ell_q \approx (\varepsilon/\kappa^3)^{-1/4}$ must depend explicitly on κ [5)].

In order to apply an analysis based on Eq. (2), we must bear in mind that this coarse-grained equation sufficiently accurately describes the superfluid velocity field on the scale over which the averaging is done. This approach cannot therefore include initial conditions similar to those commonly believed to apply in counterflow turbulence in He II if only a single scale is assumed. Such a distribution of vortices will most likely decay according to the Vinen equation [12]. It is well known and in agreement with simulations by Schwarz [13][6)] that there is a critical self-sustaining counterflow velocity, above which the turbulence is in dynamical equilibrium. According to these computer simulations and a common belief based on the experiments of Awschalom *et al* [14] this state is, at least approximately, homogeneous. If it contains just one scale, the vortex line density, L, ought to decay as $1/t$ as follows from the Vinen equation and, according to some experiments [12, 15], it most likely does.

Now let us increase the counterflow velocity U_{cf}, assuming the normal fluid velocity profile remains flat, and continue the discussion in the reference frame where the normal fluid is at rest. It is an established experimental fact that another transition (from turbulent state I to turbulent state II with distinctly different features, in accord with Tough's classification scheme [1]) occurs [1, 17, 18]. It has been a long lasting challenge to ex-

4) Possibly logarithmically corrected, see [4].

5) The relevant discussion is contained in [11]; here we only remind that the exact functional form of the spectral energy density, $\Phi(\varepsilon, k, \kappa)$, around and beyond ℓ_q cannot be written explicitly based on the dimensional analysis similar to that of Kolmogorov (see [11]), as it can contain, in principle, any function of the dimensionless combination $\varepsilon\kappa^{-3}k^{-4}$. However, the form of $\Phi(\varepsilon, k, \kappa)$ can be judged if one uses experimental data on the late decay of the grid generated turbulence [16], relevant to scales of order $\ell_{diss} \approx (\varepsilon/\nu_n^3)^{-1/4}$, where the normal fluid can be considered at rest.

6) Note, however, that these simulations have been performed using so called local induction approximation, so the non local interaction as it takes place in classical turbulence is automatically cut out.

plain the nature of this transition. We believe that the answer might be hidden in Volovik's analysis [3, 4]. As he claims, there is a crossover between what he calls the Kolmogorov and Vinen states of superfluid turbulence when

$$\mathrm{Re}_s q^2 = U_{cf} R q^2/\kappa \simeq 1 \qquad (3)$$

This result has been confirmed in the most recent theoretical work [7]. For higher counterflow velocities an analysis based on Eq. (2) is likely to be valid and therefore a range of scales between the outer scale, R and a minimum scale to which the cascade extends due to mutual friction occurs, and this minimum scale (it would be misleading to call it dissipative scale, as dissipation occurs at all scales and is more important at large ones) still exceeds the quantum scale. Within these scales the superfluid turbulence cannot be of the pure Kolmogorov type – the analysis [6, 7] clearly show that in such a case there is no dissipation and nothing stops turbulent energy from propagating to smaller and smaller scales, eventually violating the above condition of circulation at smallest scale exceeding κ. The energy spectrum thus must be of k^{-3} – type, or, at least of mutual friction–modulated $k^{-5/3}$ type – see Eq. (28) in [7]. We therefore have large superfluid eddies that strongly interact via mutual friction with the normal fluid. So far we considered the theoretical approach when the normal fluid is strictly at rest, but in He II this will become most likely violated and the normal fluid driven into a turbulent state, too[7]. In a steady counterflow the coupled turbulence similar to the towed grid coupled turbulence [19], is unlikely, as the big superfluid and normal eddies are on average taken apart by the counterflow velocity. The situation changes, however, when the heater that generates the counterflow is turned off and the turbulence decays. Big normal and superfluid eddies can match each other, there will be therefore no energy loss by mutual friction at large scales. As a result the low k part of the spectrum will change from k^{-3} type towards classical $k^{-5/3}$ Kolmogorov type. We believe that this is the reason why so called "anomalous" decay of counterflow turbulence in He II was observed in the pioneering work of Vinen [12] and later by Schwarz and Rosen [20]. In our own decay experiments [21], we have observed that the temperature gradient along the counterflow channel decays very fast when the heater is switched off, so the flow can be considered as isothermal when the second sound decay measurement is being performed [23]. We therefore expect that the decays of high Re_s counterflow turbulence and grid generated He II turbulence ought to display the same character. And, indeed, it was clearly shown in experiments, that for both towed grid generated He II turbulence [16] and high Re_s counterflow turbulence [21], most of the decay[8] of the vortex line density displays the same $t^{-3/2}$ power law. This decay law follows from the spectral decay model [16] based on the existence of the Kolmogorov 3d energy spectrum, directly shown to be present in classically generated ^{4}He turbulence both above and below the lambda point by Maurer and Tabeling [24].

The crossover to superfluid turbulence state II has been observed in channels of circular and square crossection, but not in narrow channels of high aspect ratio rectangular crossection [25]. Naturally, the transition cannot take place if the size of the sample intervenes. If some dimension of the channel is too small, its physical size limits the size of eddies.

There are many experimental data that can be used in order to probe the existence of the phase diagram (see Fig.1) suggested by Volovik [3, 4]. The recent experiment of Finne et al. [8] provides evidence for a velocity – independent transition from a laminar to a turbulent flow regime in rotating ^{3}He-B, where values of q of order unity are experimentally easily accessible.

In He II these large values of q occur very close to the lambda point, where, to our knowledge, no reliable measurements exist that can be considered in the frame of reference of the normal fluid. On the other hand, there is ample experimental data on counterflow He II turbulence at lower temperatures. However, the data on the transition into superfluid state I (Vinen state) in tubes and capillaries of various sizes cannot be reliably used here, as it is believed that below this threshold the viscous normal fluid possesses a velocity profile similar to a flow of ordinary viscous flow in a pipe. A unique frame of reference is not, therefore, provided by the normal fluid. However, Baehr et al. [26] studied the transition from dissipationless superflow to homogeneous superfluid turbulence, when both ends of the pipe were blocked by superleaks and the normal fluid inside the pipe thus remains stationary (at least on average), thereby providing this unique frame of reference. These data, spanning the temperature range 1.3 K $< T <$ 1.9 K, mark the transition from laminar flow into the Vinen state (state I) shown in Fig.1.

That the existence of the normal fluid reference frame is important can be demonstrated using the different set

[7] That the normal fluid is likely turbulent in state II of the counterflow He II turbulence is independently supported by the stability analysis of Melotte and Barenghi [22].

[8] After the energy containing length scale becomes saturated by the size of the channel [16].

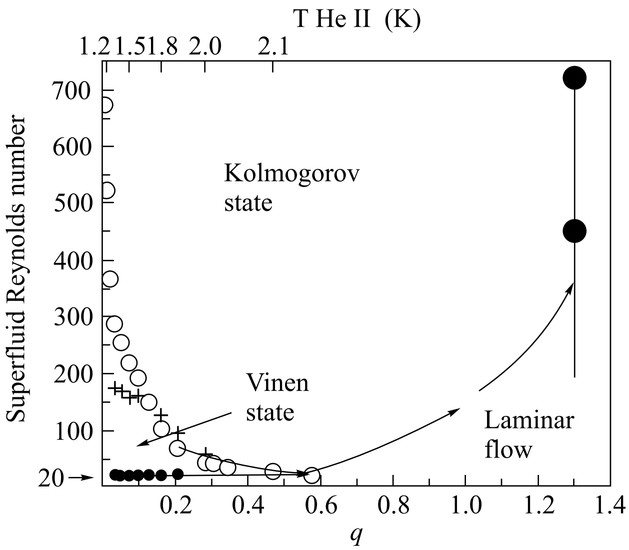

Fig.1. The observed flow phase diagram of He II and ^{3}He-B superfluids in the unique frame of reference where the normal fluid is at rest. The abscissa, q, represents the inverse of the Reynolds number for superflow (for convenience the corresponding temperature in He II is indicated on the upper axis), while the ordinate, Re_s, represents the strength of circulation at the outer scale of the flow in units of κ. The small filled circles represent the onset of turbulent state I in pure superflow of He II when the motion of the normal fluid was inhibited by superleaks [26]; the crosses [17] and open circles [28] mark the transition from state I into state II for counterflowing He II. The two big filled points mark approximately the region where the onset of superfluid turbulence has been observed by various methods of vortex loop injection into rotating ^{3}He-B in the vortex-free Landau state (for $\Delta T \approx 0.05\ T_c$ around 0.6 T/T_c at 29 bar, see Fig.3 in [8])

of data of Baehr and Tough [27] obtained with essentially identical setup but for flows where at fixed temperature the normal and superfluid velocities can be varied independently. If the normal fluid flow profile were flat, one would expect that the critical counterflow velocity for transition into turbulent state I, simply as a consequence of the Galilean invariance, stays unchanged. This does not happen, as the observed critical counterflow velocity increases with the imposed averaged normal fluid velocity, supposedly due to the fact that its flow profile is not flat.

Various counterflow experiments clearly display the transition from state I (Vinen) into state II (Kolmogorov) – the signature is pronounced on temperature and pressure difference versus heat input dependencies. We use here the data of Ladner, Childers and Tough [17] (their Table I), assuming that in state I the normal fluid profile is flat, again providing the unique frame of reference with the normal fluid at rest. Let us point out that this transition into a different flow regime is accompanied with a pronounced increase of fluctuations [18], characteristic of phase transitions. The data [17] also clearly show that on increasing the temperature the difference in counterflow velocity between state I and II transitions decreases and around 2 K they become indistinguishable.

As another set of experimental data marking the state I – state II transition we have used the thermal conduction measurements of He II in tubes of various diameters of Chase [28]. We have scanned the available experimental data and show in Fig.2 that they collapse onto a single

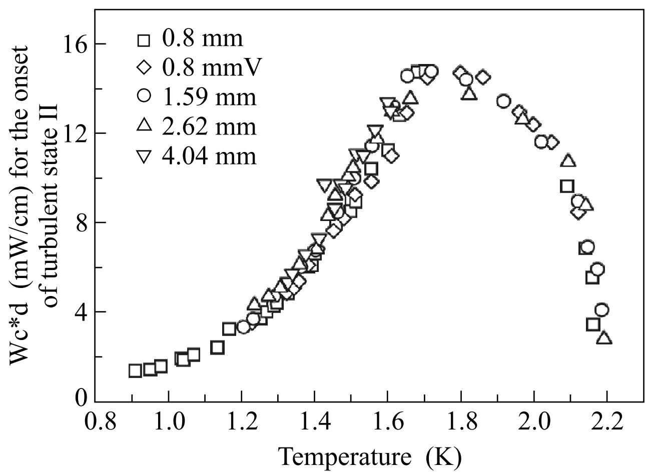

Fig.2. Product of the critical heat input per unit area, Wc, times the inner diameter of the used tube, d, marking the onset of the turbulent state II versus temperature [28] (two different experimental methods have been used for a tube with $d = 0.8$ mm). The data obtained with tubes of various d as indicated collapse onto a single curve. Assuming that $U_{cf} \propto Wc$, these sets of data demonstrate that the onset of turbulent state II occurs at any temperature at a critical value of Re_s, shown in Fig.1

curve if Reynolds number scaling is applied. The open circles in Fig.1 correspond to the onset of state II.

We emphasize that the procedure used to acquire the data points shown in the flow diagram is probably not very accurate for several reasons, such as different temperature scales or uncertainty in values of q, and more work is needed to map it out accurately. We believe, however, that the essential physics is displayed clearly and that Fig.1 strongly supports the ideas underlying the physical problem of superfluid turbulence.

Let us comment here on the apparent disagreement in the zero temperature limit between this phase flow diagram (Vinen state preferred for any finite Re_s) and the computer simulations of Araki et al. [29] strongly indicating the presence of Kolmogorov scaling . Although these simulations are performed in the zero temperature limit in the sense that there is no normal fluid, simulation introduces a cutoff at the scale of the grid used for simulations, in that all vortex rings or loops smaller than this size are removed from the flow. This effectively introduces an artificial dissipative mechanism at a prescribed length scale. We believe that any dissipative mechanism acting at some small length scale (such as a Kelvin wave cascade with subsequent phonon emission [30]) leads to a Kolmogorov cascade in the continuum approximation, as the assumptions for it are only that there is a range of scales where dissipation is unimportant, that the form of the energy spectrum only depends on k, and that the total energy decay rate $\varepsilon = -dE/dt$ is independent of k. Dimensional analysis then leads to the energy spectrum of the Kolmogorov form. The physical mechanism of the dissipation is unimportant, so long as it acts only on small scales.

In practice, there could be a crossover from the mutual friction dissipation regime into a different one, probably based on vortex wave irradiation. The governing equation (2) will have to be altered accordingly, and similar analysis repeated.

To conclude, we show that the extraordinary fluid properties of quantum fluids give rise to the flow diagram suggested by Volovik [3], containing two distinctly different turbulent flow regimes. These can most likely be identified with the puzzling turbulent states I and II according to the classification scheme of Tough [1]. We emphasize that the energy spectrum in what is called Kolmogorov state does not have the classical roll-off exponent -5/3.

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1. J. T. Tough, Superfluid turbulence, in *Prog. in Low Temp. Phys.* Vol.VIII, North-Holland, Amsterdam, 1982.
2. D. D. Awschalom and K. W. Schwarz, Phys. Rev. Lett. **52**, 49 (1984).
3. G. E. Volovik, JETP Lett. **78**, 553 (2003).
4. G. E. Volovik, arXiv:cond-mat/0402035.
5. E. B. Sonin, Rev. Mod. Phys. **59**, 87 (1987).
6. W. F. Vinen, *The theory of quantum grid turbulence in superfluid* ^{3}He-B, submitted to Phys. Rev. **B**.
7. V. S. L'vov, S V. Nazarenko, and G. E. Volovik, Pis'ma ZhETF **80**, 546 (2004).
8. A. P. Finne et al., Nature **424**, 1022 (2003).
9. R. J. Donnelly and C. F. Barenghi, J. Phys. Chem. Data **27**, 1217 (1998).
10. T. D. C. Bevan et al., J. Low Temp. Phys. **109**, 423 (1997).
11. L. Skrbek, J. J. Niemela, and K. R. Sreenivasan, Phys. Rev. **E64**, 067301 (2001).
12. H. E. Hall and W. F. Vinen, Proc. Roy. Soc. **A238**, 204 (1956); W. F. Vinen, Proc. Roy. Soc. **A240**, 114 (1957); **A242**, 489 (1957).
13. K. W. Schwarz, Phys. Rev. **B38**, 2398 (1988).
14. D. D. Awschalom, F. P. Milliken, and K. W. Schwarz, Phys. Rev. Lett. **55**, 1372 (1984).
15. F. P. Milliken and K. W. Schwarz, Phys. Rev. Lett. **48**, 1204 (1982).
16. L. Skrbek, J. J. Niemela, and R. J. Donnelly, Phys. Rev. Lett. **85**, 2973 (2000).
17. D. R. Ladner, R. K. Childers, and J. T. Tough, Phys. Rev. **B13**, 2918 (1976).
18. C. P. Lorenson, D. Griswold, V. U. Nayak, and J. T. Tough, Phys. Rev. Lett. **55**, 1494 (1985).
19. W. F. Vinen and J. J. Niemela, J. Low Temp. Phys. **128**, 167 (2002).
20. K. W. Schwarz and J. R. Rosen, Phys. Rev. Lett. **66**, 1898 (1991); Phys. Rev. **B44**, 7563 (1991).
21. L. Skrbek, A. V. Gordeev, and F. Soukup, Phys. Rev. **E67**, 047302 (2003).
22. D. J. Melotte and C. F. Barenghi, Phys. Rev. Lett. **80**, 4181 (1998).
23. A. V. Gordeev, T. V. Chagovets, F. Soukup, and L. Skrbek, *Decaying Counterflow Turbulence in* He II, Journal of Low. Temp. Phys., in print
24. J. Maurer and P. Tabeling, Europhys. Lett. **43**, 29 (1998).
25. C. P. Lorenson, D. Griswold, V. U. Nayak, and J. T. Tough, Phys. Rev.**B23**, 1494 (1985).
26. M. L. Baehr, L. B. Opatowsky, and J. T. Tough, Phys. Rev. Lett. **51**, 2295 (1983).
27. M. L. Baehr and J. T. Tough, Phys. Rev. Lett. **53**, 1669 (1984).
28. C. E. Chase, Phys. Rev.**127**, 361 (1962);**131**, 1898 (1963).
29. T. Araki, M. Tsubota, and S. K. Nemirovskii, Phys. Rev. Lett. **89**, 145301 (2002).
30. W. F. Vinen, Phys. Rev. **B61**, 1410 (2000).