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On thermodynamic and quantum fluctuations of cosmological constant

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We discuss from the condensed-matter point of view the recent idea that the Poisson fluctuations of cosmological constant about zero could be a source of the observed dark energy [1, 2]. We argue that the thermodynamic fluctuations of Λ are much bigger. Since the amplitude of fluctuations is $\propto V^{-1/2}$ where V is the volume of the Universe, the present constraint on the cosmological constant provides the lower limit for V, which is much bigger than the volume within the cosmological horizon.

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Recently the following approach to the solution of the cosmological problem has been suggested: the average value of the cosmological constant is zero, $\langle\Lambda\rangle = 0$, due to the ability of quantum micro structure of spacetime to readjust itself and absorb bulk vacuum energy densities; the observed dark energy is provided by residual quantum fluctuations about zero which simulate a "small" cosmological constant [1, 2].

Here we discuss this scenario using the condensed-matter experience. In the condensed-matter example of 'quantum gravity', the 'trans-Planckian' physics is well known, while gravity naturally emerges in the low-energy corner together with gauge fields and chiral fermions (see [3]). This working model of 'quantum gravity' provides us with some criteria for selection of theories: those theories of quantum vacuum are favourable which are consistent with its condensed-matter analog. Here we apply this test to the theory of the fluctuating cosmological constant. We show that this idea is consistent with the condensed-matter 'oracle', but the amplitude of the fluctuations is in contradiction.

The first assumption made in papers [1, 2], that the average value of Λ is zero, $\langle\Lambda\rangle = 0$, is certainly consistent with the condensed-matter 'quantum gravity', where the same cosmological constant problem arises and is resolved. As in our quantum vacuum, the naive estimation of the energy density of the condensed matter in its ground state gives the 'natural' characteristic value for the induced cosmological constant

$$\Lambda_0 \sim \frac{E_{\rm Pl}^4}{\hbar^3 c^3}. \qquad (1)$$

Here $E_{\rm Pl}$ is the corresponding ultraviolet energy cut-off of atomic scale, which plays the role of the Planck energy scale. However, the microscopic (atomic) phyiscs is well known which allows us to make exact calculations of the ground state energy. One obtains that for the equilibrium ground state (vacuum) in the absence of excitations above the vacuum (matter), the energy density of the system isolated from the environment is exactly zero. In the language of gravity, this means that $\langle\Lambda\rangle = 0$. The contribution Λ_0 from sub-Planckian and Planckian modes of the quantum vacuum is exactly cancelled by the microscopic trans-Planckian (atomic) degrees of freedom without any fine-tuning.

This remarkable result follows from the general thermodynamic analysis applied to the quantum vacuum, which utilizes the Gibbs-Duhem relation (see e.g. Ref. [4]). The same analysis demonstrates that in the presence of matter, Λ is more or less on the order of matter density or of other perturbations of the vacuum [3, 4].

If the perturbations of the vacuum are small, the cosmological constant induced by these perturbations may become smaller than thermodynamic or quantum fluctuations of the cosmological constant about zero. Then we enter the regime, which has been considered in Refs. [1, 2]. According to this suggestion the pesently observed dark energy is nothing but the fluctuation of the cosmological constant. To test this idea we extend our thermodynamic analysis incorporating the thermodynamic and quantum fluctuations of Λ.

The thermodynamic fluctuations of cosmological constant Λ, and thus of the vacuum pressure $P_{\rm vac} = -\Lambda$, correspond to the thermodynamic fluctuations of pressure in condensed matter system, say in a quantum liquid, when it is close to its ground state (vacuum). In quantum liquids, assuming that the liquid is in equilib-

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rium without environment, the average pressure is zero $\langle P\rangle = 0$. This corresponds to

$$\langle\Lambda\rangle = -\langle P_{\rm vac}\rangle = 0 \qquad (2)$$

in the equilibrium vacuum. The thermodynamic fluctuations of pressure are given by the general thermodynamic relation [5]:

$$\langle P^2\rangle_{\rm Thermal} = -T\left(\frac{\partial P}{\partial V}\right)_S , \qquad (3)$$

where S is entropy, T is temperature, and V is the volume of the system. In quantum liquids, the Eq. (3) is expressed through the mass density of the liquid ρ and the speed of sound s:

$$\langle P^2\rangle_{\rm Thermal} = \frac{\rho s^2 T}{V}. \qquad (4)$$

Applying this to the vacuum, we use the fact that in quantum liquids, the quantity ρs^2 is on the order of the characteristic energy density of the liquid. That is why in the quantum vacuum, this would correspond to the natural value of the vacuum energy density, i.e. to the natural value of the cosmological constant expressed through the Planck energy scale in Eq. (1):

$$\rho s^2 \equiv \Lambda_0 \sim \frac{E_{\rm Pl}^4}{\hbar^3 c^3}, \qquad (5)$$

and we have the following estimation for the amplitude of thermal fluctuations of the cosmological constant:

$$\langle\Lambda^2\rangle_{\rm Thermal} = \langle P_{\rm vac}^2\rangle_{\rm Thermal} = \Lambda_0 \frac{T}{V}. \qquad (6)$$

Thus the temperature of the Universe influences not only the matter degrees of freedom (and thus the geometry of the Universe), but it also determines the thermal fluctuations of cosmological constant. This is an important lesson from the condensed matter: thermal fluctuations of the vacuum pressure are determined by vacuum degrees of freedom, and not by much smaller matter degrees of freedom as one can naively expect.

Note that in Eq. (6), Λ_0 is the contribution to the vacuum energy density and to cosmological constant, which comes from the Planck scale modes in the quantum vacuum. But both in quantum liquids and in the quantum vacuum, the total contribution of all the modes to the energy density is zero if the vacuum is in complete equilibrium, $\langle\Lambda\rangle = 0$. The tiny contribution to the vacuum energy density comes from a small disbalance caused by perturbations of the vacuum state, and here we discuss the disbalance produced by the "hydrodynamic" fluctuations of the vacuum.

From Eq. (6) we can get the estimate for the quantum fluctuations of the vacuum pressure and cosmological constant, which are important when temperature becomes comparable to the distance between the quantum levels in the finite volume, i.e. when $T \sim \hbar c/V^{1/3}$. Substituting this T into Eq. (6) one obtains the amplitude of quantum fluctuations of Λ:

$$\langle\Lambda^2\rangle_{\rm Quantum} = \langle P_{\rm vac}^2\rangle_{\rm Quantum} = \Lambda_0 \frac{\hbar c}{V^{4/3}}. \qquad (7)$$

Let us apply this first to the vacuum in the closed Einstein Universe with the volume of the 3D spherical space $V = 2\pi R^3$. This gives the following estimation for the amplitude of quantum fluctuations of Λ:

$$\sqrt{\langle\Lambda^2\rangle}_{\rm Quantum} \sim \frac{E_{\rm Pl}^2}{\hbar c R^2}. \qquad (8)$$

It is comparable to the mean value of Λ in Einstein Universe. For example, in the cold Universe one has $\langle\Lambda\rangle = E_{\rm Pl}^2/8\pi\hbar c R^2$, while in the hot Universe $\langle\Lambda\rangle = 3E_{\rm Pl}^2/16\pi\hbar c R^2$. Moreover, in any reasonable Universe the temperature is much bigger than the quantum limit, $T \gg \hbar c/R$, and thus the thermodynamic fluctuations of Λ highly exceed the value required for the construction of the closed solution. This means that thermodynamic fluctuations would actually destroy the Einstein solution. However, we must note that in the thermodynamic derivation used it was assumed that the closed Universe is in a thermal (but not in the dynamical) contact with the 'environment', i.e. the Universe is immersed in a thermal reservoir at a fixed temperature, say, as a brane immersed in the higher-dimensional world. The Einstein Universe under these conditions has been studied in Ref. [6]. For the really closed system, i.e. without any contact with the environment, such fluctuations are suppressed due to the energy conservation.

Let us now turn to our Universe which is spatially flat and thus is formally infinite. What volume is relevant for the thermodynamic and quantum fluctuations? If we suppose that $V \sim R_h^3$, where R_h is on the order of the scale of the cosmological horizon, then for the amplitude of the quantum fluctuations of Λ one obtains just what has been suggested in Refs. [1, 2]:

$$\sqrt{\langle\Lambda^2\rangle}_{\rm Quantum} \sim \frac{E_{\rm Pl}^2}{\hbar c R_h^2}. \qquad (9)$$

For the same estimation of the vacuum energy density using different approach see Ref. [7].

Equation (9) gives the correct order of magnitude for Λ in the present Universe. However, again this is negligibly small compared to the amplitude of thermal fluctuations of Λ at any resonable temperature of the

Universe. Assuming, for example, that the temperature of the Universe coincides with the temperature $T_{\rm CMB}$ of the cosmic microwave background radiation, one obtains the following amplitude:

$$\sqrt{\langle\Lambda^2\rangle}_{\rm CMB} \sim \frac{E_{\rm Pl}^2 T_{\rm CMB}^{1/2}}{(\hbar c)^{3/2} R_h^{3/2}} \sim$$
$$\sim 10^{14}\frac{E_{\rm Pl}^2}{\hbar c R_h^2} \gg \sqrt{\langle\Lambda^2\rangle}_{\rm Quantum}. \qquad (10)$$

Such a big value of Λ is certainly in contradiction with observations. This means that we cannot use the volume inside the horizon as the proper volume for estimation of the quantum and thermal fluctuations of Λ. The reason is that we are interested not in the fluctuations of Λ from one Hubble volume to another, but in the influence of the Λ term on the dynamics of the whole Universe. Thus instead of the Hubble volume R_h^3 one must use the real volume of the Universe, which is not limited by the cosmological horizon.

From the condition that thermal fluctuations of Λ at $T_{\rm CMB}$ must be smaller than the experimental upper limit, $\sqrt{\langle\Lambda^2\rangle}_{\rm CMB} < \Lambda_{\rm exp}$, one can estimate the lower limit for the size of the Universe:

$$V > \frac{T_{\rm CMB}\Lambda_0}{\Lambda_{\rm exp}^2} \sim \frac{T_{\rm CMB}}{\hbar H} R_h^3 \sim 10^{28} R_h^3, \qquad (11)$$

where $H \sim c/R_h$ is the Hubble constant. Note that considering distances much bigger than the size of the horizon we do not sacrifice the usual notion of causality. The "hydrodynamic" fluctuations of the vacuum are independent in different volumes of the space if these volumes are separated by distance $|\mathbf{r}_1 - \mathbf{r}_2| > \hbar c/T$:

$$\langle\delta P(\mathbf{r}_1)\delta P(\mathbf{r}_2)\rangle_{\rm Thermal} = \Lambda_0 T\delta(\mathbf{r}_1 - \mathbf{r}_2),$$
$$|\mathbf{r}_1 - \mathbf{r}_2| > \frac{\hbar c}{T}. \qquad (12)$$

Since $T \gg \hbar c/R_h$, the correlations decay at distances much smaller than R_h, and thus we do not need correlations across the horizon.

The expanding Universe is certainly not in equilibrium. The noise caused by deviations from the equilibrium adds to the estimated fluctuations and may change the effective temperature. Moreover, Λ in Eq. (9) depends on time in expanding Universe, and this dependence is inconsistent with observations. Supernovae data show that the dark energy density is either constant with time [8], or it still may depend on time, but rather weakly [9]. In the latter scenario, the dark energy density is constant on average up to redshifts of 0.5 and growing to the past for larger redshifts. But it still remains significantly less than the matter energy density.

Our thermodynamic analysis here cannot consider the dynamics of the Universe. However, the fact that the dark energy density is not very far from the density of dark matter demonstrates that the Universe is not very far from equilibrium [4], and thus our estimations are not highly distorted by expansion. To incorporate fluctuations of the cosmological constant and its time dependence in the real Universe, the Einstein equations must be modified; otherwise both the fluctuations and time-dependence are prohibited by Bianchi identities. But even in this case the condensed-matter 'quantum gravity' can be useful, since it provides us with the general scheme of how the dissipation is introduced. Following this scheme one can incorporates the extra terms the Einstein equations which contain the time derivative of Λ [10]. The modification of the Einstein equations was also suggested in [2], but it represents the particular case of the two-parametric modification suggested in [10]. The two parameters introduced in Ref. [10] and considered as phenomenological fitting parameters, can in principle fit the real behavior of Λ in expanding Universe.

And the last comment concerns the emergent gravity and relativistic quantum field theory in the condensed-matter systems in general. The usage of fermionic condensed matter as a primer of emergent physics does not mean that we necessarily return to a view of spacetime as an absolute background structure incorporating absolute simultaneity. The emergence is based on the general topological properties of fermionic vacua in momentum space (see Chapter 8 of [3]) which are not sensitive to the background geometry: the background can be Galilean, Lorentzian, or something else. The only requirement for the application of the topology in momentum space is the invariance of the vacuum under translations, so that the Green's function can be represented in terms of the 4-momentum. If the Universe is inhomogeneous, the 4-momentum and thus the momentum space topology are well determined in the quasiclassical limit, i.e. for the wavelengths much smaller than the characteristic size of the Universe. This is enough for the construction of the universality classes of quantum vacua. For one of the two important universality classes, the main physical laws of Standard Model and gravity gradually emerge in the low energy corner irrespective of the background. If the background is originally non-Lorentzian, the information on this background is lost in the low-energy corner where the effective Lorentzian space-time gradually arises.

It was recently suggested from the consideration of the highest energy cosmic rays observed that Lorentzian invariance is probably more fundamental than all the other physical laws, since it persists even well beyond the Planck energy scale $E_{\rm Pl}$ [11]. This is an encouraging fact for the emergent physics. According to Bjorken [12], the emergence can explain the high precision of the present physical laws only if there is an extremely small expansion parameter. This small parameter can be the ratio of the energy scales: $E_{\rm Pl}/E_{\rm Lorentz}$, where $E_{\rm Lorentz} \gg E_{\rm Pl}$ is the energy scale where the Lorentz invariance is violated. In the effective action for the gauge and gravity fields obtained by integration over fermions, the integration region is limited by the Planck energy cut-off. If $E_{\rm Pl} \ll E_{\rm Lorentz}$, then integration is concentrated in the fully relativistic region, where fermions are still very close to Fermi points and thus obey the effective gauge invariance and general covariance which follow from the topology of a Fermi point (see Chapter 33 of [3]). As a result the effective bosonic action also becomes invariant, and the precision of the emergent physical laws is determined by some power of $E_{\rm Pl}/E_{\rm Lorentz}$.

This is the reason why we assume that Lorentzian invariance is obeyed at the microscopic level. The Lorentzian invariance implies that the analog of the vacuum speed of sound s in Eq. (4) coincides with the speed of light c. As a result the thermal fluctuations of Λ are determined by the "bare" vacuum energy Λ_0 according to Eq. (5).

In conclusion, from the condensed-matter analog of 'quantum gravity' it follows that the main idea of Refs. [1, 2], that fluctuations $\sqrt{\langle\Lambda^2\rangle}$ of the cosmological cosnstant may dominate over its mean value $\langle\Lambda\rangle$, is correct. However, it appears that the thermal fluctuations of Λ at any reasonable temperature of the Universe are much bigger than the 'quantum' Poisson-type fluctuations suggested in Refs. [1, 2]. If this analogy is correct, the thermal fluctuations of Λ pose the limit on the size of the Universe, which must be by at least 9 orders of magnitude bigger than the size of the cosmological horizon.

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