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The mesoscopic chiral metal-insulator transition

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Sharp localization transitions of chiral edge states in disordered quantum wires, subject to strong magnetic field, are shown to be driven by crossovers from two- to one-dimensional localization of bulk states. As a result, the two-terminal conductance is found to exhibit at zero temperature discontinuous transitions between *exactly* integer plateau values and zero, reminiscent of first order phase transitions. We discuss the corresponding phase diagram. The spin of the electrons is shown to result in a multitude of phases, when the spin degeneracy is lifted by the Zeeman energy. The width of conductance plateaus is found to depend sensitively on the spin flip rate $1/\tau_s$.

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The high precision of the quantization of the Hall conductance of a two dimensional (2D) electron system (2DES) in a strong magnetic field [1] is known to be due to the binding of electrons to localized states in the bulk of the 2DES. Thereby, a change of electron density does not change the Hall conductance [2 – 4]. The localization length in tails of Landau bands is very small, of the order of the cyclotron length $l_{\rm cyc} = v_F/\omega_B = \sqrt{2n+1}l_B$. It increases towards the centers of the Landau bands, $E_{n0} = \hbar\omega_B(n+1/2)$ $(n = 0, 1, 2, \dots)$, with $\omega_B = eB/m^*$ the cyclotron frequency (e elementary charge, m^* effective mass), v_F the Fermi velocity, and $l_B^2 = \hbar/eB$ defines the magnetic length. In an *infinite* 2DES in perpendicular magnetic field, the localization length at energy E diverges as $\xi \sim |E - E_{n0}|^{-\nu}$, reminiscent of 2nd order phase transitions. The critical exponent ν is found numerically for the lowest two Landau bands, $n = 0, 1$, to be $\nu = 2.33 \pm 0.04$ for spin-split Landau levels [5, 6]. Analytical [7] and experimental studies [8] are consistent with this value. In a *finite* 2DES, a region of extended states should exist in the centers of disorder broadened Landau-bands. These states extend beyond the system size L. The width of these regions is given by $\Delta E = (l_{\rm cyc}/L)^{1/\nu}\Gamma$, where $\Gamma = \hbar(2\omega_B/\pi\tau)^{1/2}$ is the band width, with elastic scattering time τ.

In quantum Hall bars of finite width, there exist in addition edge states with energies lifted by the confinement potential above the energies of centers of bulk Landau bands, E_{n0} [4]. Previously, there has been a considerable interest in the study of mesoscopically narrow quantum Hall bars [9], with emphasis on conductance fluctuations [10, 11], edge state mixing [12 – 15], breakdown of the quantum Hall effect [16], and quenching of the Hall effect due to classical commensurability effects [17]. It is known that in the presence of white noise disorder the edge states do mix with the bulk states when the Fermi energy is moved into the center of a Landau band. It had been suggested that this might result in localization of edge states [14, 15]. In this paper, we show that this is indeed the case. In particular, at zero temperature the two-terminal conductance of a quantum wire in a magnetic field exhibit for uncorrelated disorder and hard wall confinement discontinuous transitions between integer plateau values and zero, Fig.1.

The localization length ξ in a 2DES with broken time reversal symmetry is expected from scaling theory [19 – 22] and numerical scaling studies [23, 24] to be

$$\xi_{\rm 2D} = l_0 e^{\pi^2 g^2}, \qquad (1)$$

depending exponentially on g, the 2D conductance parameter per spin channel; l_0 is the short distance cutoff, which is the elastic mean free path $l = 2g(B = 0)/k_{\rm F}$ ($k_{\rm F}$ Fermi wave number) at moderate magnetic fields, $b \equiv \omega_B\tau < 1$. For stronger magnetic fields, $b > 1$, l_0 crosses over to the cyclotron length $l_{\rm cyc}$. g exhibits Shubnikov-de-Haas oscillations as function of magnetic field for $b > 1$. Maxima occur when the Fermi energy is in the center of Landau bands. Thus, the localization length increases strongly from band tails to band centers, even when the wire width L_y is too narrow to allow delocalization of bulk states. For uncorrelated impurities, within self consistent Born approximation [25], the maxima are given by $g(E = E_{n0}) = (2n + 1)/\pi = g_n$. Thus, $\xi_{\rm 2D}(E_{n0}) = l_{\rm cyc}\exp(\pi^2 g_n^2)$ are macroscopically large in centers of higher Landau bands, $n > 1$ [6, 26]. However, when the width of the system L_y is smaller

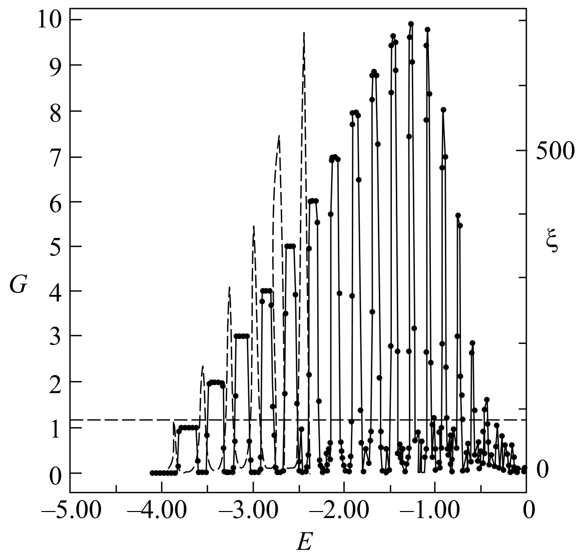

Fig.1. Solid curve, left axis: two-terminal conductance of a wire (width $L_y = 80a$, length $L = 5000a$, hard wall confinement) in units of $G_0 = e^2/h$ as function of energy E in units of t (hopping amplitude in the tight binding model). Disorder strength, $W = 0.8t$. There are $x = 0.025$ magnetic flux quanta through an elementary cell a^2. Dashed curve, right axis: bulk localization length ξ in units of a as function of E, from transfer matrix method for wire of identical properties but periodic boundary conditions and $L\max = 100000a$. Straight dashed line: $L_y = 80a$

than ξ_{2D}, localization is expected to behave quasi-1D. In other words, electrons in centers of Landau bands can diffuse between the edges of the system, but are localized parallel to the edges if $L_y < \xi_{2D}$. The quasi-1D localization length is known to depend only linearly on g. In a magnetic field, when time reversal symmetry is broken, it is [27−29]

$$\xi_{1D} = 2g(B)L_y. \qquad (2)$$

There is a crossover from 2D to 1D localization as the Fermi energy is moved from tails to centers of Landau bands. Performing a renormalization of the wire conductance, one obtains for the localization length [30],

$$\xi^2 = 4L_y^2 g^2 - \frac{2L_y^2}{\pi^2}\ln\left[\frac{1+(L_y/l_0)^2}{1+(L_y/\xi)^2}\right]. \qquad (3)$$

Its solution shows a crossover between the quasi-1D- and the 2D-behaviour, Eqs. (2) and (1), respectively.

The conductance per spin channel, $g(b) = \sigma_{xx}(B)/\sigma_0$, is given by the Drude formula $g(b) = g_0/(l + b^2)$, ($g_0 = E\tau/\hbar$, $b = \omega_B\tau$) for weak magnetic field, $b < 1$. For $b > 1$, when the cyclotron length $l_{\rm cyc}$ is smaller than the mean free path l, disregarding the overlap between Landau bands, g is obtained in SCBA [25],

$$g(B) = \frac{1}{\pi}(2n+1)\left(1 - \frac{(E_{\rm F}-E_n)^2}{\Gamma^2}\right), \qquad (4)$$

for $|E - E_n| < \Gamma$. One obtains the localization length for $b > 1$ and $|\epsilon/b - n - 1/2| < 1$ by inserting g, Eq. (4) into Eq. (3). It oscillates between maximal values in centers of Landau bands, and minimal values in band tails (Fig.2). For $n > 1$, one finds in band centers,

$$\xi_n = \frac{2}{\pi}(2n+1)L_y\left[1 - \frac{\ln\sqrt{1+(l_y/l_{\rm cyc})^2}}{(n+1/2)^2}\right]^{1/2}. \qquad (5)$$

In the center of the lowest Landau band ($n = 0$), Eq. (2) gives a value $\xi_0(B) \approx (2/\pi)L_y$, smaller than L_y. There, the localization is 2D and the topological term [34] is effective, leading to criticality, and diverging localization lengths. In a wire of finite width, L_y the localization length ξ saturates to the critical value $\xi_{\rm crit} \approx 1.2L_y$ [5,6] larger than L_y. Comparison with ξ_n, Eq. (5), shows that the noncritical quasi-1D localization length exceeds $\xi_{\rm crit}$ in all but the lowest Landau bands.

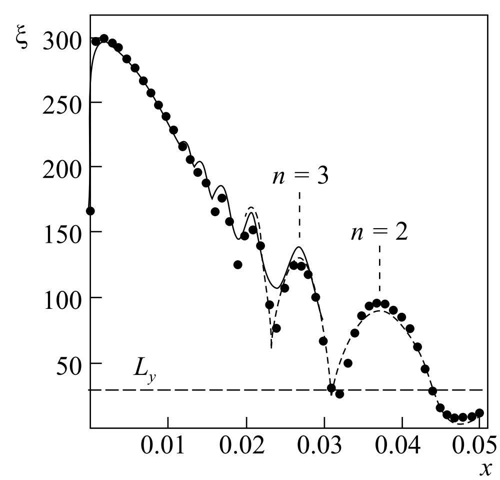

Fig.2. The localization length, ξ, as obtained by inserting $g(B)$ in 2nd order Born approximation in Eq. (3), including a summation over all Landau levels (full line). Broken line: ξ, with $g(B)$ in SCBA, neglecting overlap between Landau bands, Eq. (4). The result of transfer matrix calculations is plotted (dot size: numerical error $\approx 1\%$) for periodic boundary conditions [31] as function of magnetic flux through unit cell $x = a^2/2\pi l_B^2$. Dimensionless conductance parameter per spin channel is $g(B = 0) = 5.1$. At weak magnetic fields doubling of ξ due to breaking of time reversal symmetry is seen, in good agreement with the analytical crossover formula (full line) [30, 32]. Horizontal dashed line: $L_y = 30a$

If all states in centers of higher Landau bands are localized along the wire, the question arises if there exist extended states in the quantum Hall wire at all. Consider an annulus, with a circumference larger than the localization length in the center of a Landau band. When a magnetic flux pierces the annulus, localized states are unaffected. Guiding centers of states which extend around the annulus do shift in position and energy, however [4]. In a confined wire there exist chiral edge states, which extend around the annulus. A magnetic flux change moves these states down and up inner and outer edges, respectively. As shown above, in the middle of the Landau band, the electrons can diffuse freely from edge to edge, but are localized along the annulus with $\xi > L_y$. As a consequence, when adiabatically changing the magnetic flux, an "edge state" has to move from the inner to the outer edge, since it cannot enter the band of localized states. This fact has been interpreted as proof for the existence of an extended bulk state, extending around the annulus and between edges, at the energy E_m, with $\xi(E_m) = L_y$ [4]. In the following, we show that rather a transition from extended chiral edge states to localized states occurs at these energies, E_m.

Using the transfer matrix method [24], we have calculated the localization length as function of energy E in a tight binding model of a disordered quantum wire in perpendicular magnetic field, with periodic boundary conditions, Fig.1 (dashed curve, right axis). Indeed, its maxima are seen to increase linearly with energy E in agreement with Eq. (5). The transfer matrix result for the 2-terminal conductance G [35] is shown in Fig. 1 (solid curve, left axis) for a wire of identical properties, but hard wall boundary conditions and finite length L.

We verify that the condition $\xi(E_{m,p}) = L_y$, yields the energies $E_{m,p}$, $p = \pm$, at which m edge states mix, and transitions from the quasi-1D chiral metal to an insulator occur, as signaled by sharp jumps of G in Fig.1. Here, $m = n$ when this energy is above the bulk energy of the n-th Landau band, $p = +$, and $m = n - 1$ when it is below it, $p = -$. For $\xi < L_y$, backscattering between edges is exponentially suppressed and the localization length of edge states increases exponentially as $\xi_{\text{edge}} = \xi \exp(L_y/\xi)$. These results are summarised in the phase diagram, Fig.3a, where the value of G, in units of e^2/h, is given as function of wire width L_y and energy E in units of $\hbar\omega_B$. We find that $G = m$, where m is the number of extended edge states. When $L_y \le l_{\text{cyc}} \propto \sqrt{2n+1}$, all edge states are localized, and all conductance plateaus collapse, $G = 0$, as seen in Fig.1, close to the middle of the band, $E = 0$.

Taking into account the spin, the Zeeman splitting $E_n^+ - E_n^- = g_z \mu_B B$, lifts the spin degeneracy, where

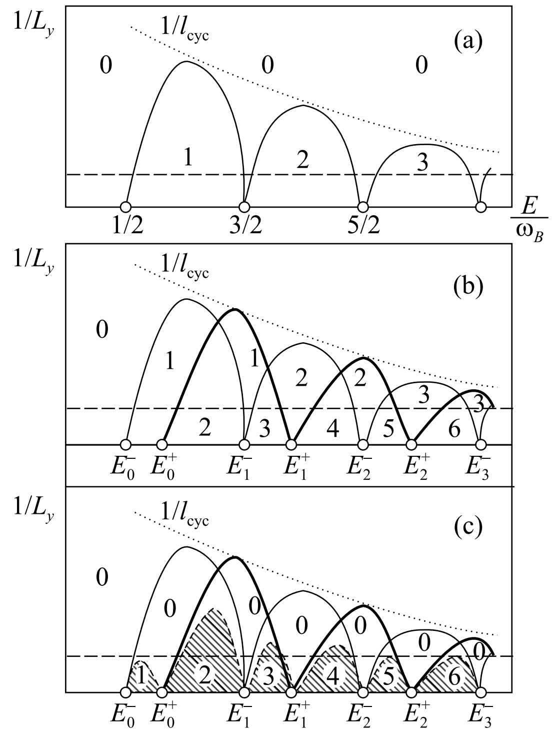

Fig.3. Schematic phase diagram of a quantum Hall wire, for $L \gg L_y$. The full lines indicate jumps between integer plateau values of conductance G, in units of e^2/h, as denoted by integers, (a) Spinless electrons, (b) electrons with spin and Zeeman–splitting $(E_n^+ - E_n^-)/\hbar\omega_B = g_Z/2$ without spin flip, and (c) with strong spin flip rate. Dotted line: inverse cyclotron length, $1/l_{\text{cyc}}$. For a particular value of inverse width $1/Ly$ (dashed line) a sequence of conductance plateaus similar to Fig.1 is obtained. In the limit of $1/L_y \to 0$ there are delocalization critical points of bulk states (circles)

g_z is the material dependent Zeeman-g-factor, and μ_B Bohr's magneton. Without spin flip scattering edge states of different spins are not mixed. The phase diagram, Fig.3b, is then a superposition of two phase diagrams, Fig.3a, for each spin. There are phases where the conductance is equal to the total number of edge channels, $G = m^+ + m^-$, when the bulk localization length of electrons ξ^σ, does not exceed the wire width, $\xi^\sigma < L_y$, for either spin $\sigma = \pm$. When, $\xi^\sigma > L_y$ for one spin σ only, the conductance is carried by the number of edge states with opposite spin, $G = m^{-\sigma}$, only. If $\xi^\sigma > L_y$ for both $\sigma = \pm$, the conductance vanishes, $G = 0$. With strong spin flip scattering, all edge state mix with bulk states for $\xi^\sigma > L_y$, yielding $G = 0$. There are conductance plateaus equal to the total number of edge states, $G = m^+ + m^-$, only if $\xi^\sigma < L_y$ is fullfilled for both spins, $\sigma = \pm$ (Fig.3c). Thus, both the sequence and width of conductance plateaus are sensitive measures of the spin flip scattering rate $1/\tau_s$, due to electron-electron interaction, spin-orbit interaction, or scattering from nuclear spins [36]. Furthermore, the enhancement of the g_Z-factor above its bulk value due to the exchange interaction depends on the Fermi energy [38]. Accordingly, the width of the conductance plateaus changes with energy. In a real sample there exists a slowly varying potential disorder which can stabilize edge states against mixing with bulk states [14]. When the confinement potential is varying slowly on the magnetic length scale l_B, the energies $E_{m,p}$ are expected to split into m energies, at which edge states mix one by one with bulk states, accompanied by steps of heights 1 in conductance G. Both potentials can be effectively modified by the Coulomb interaction as obtained by a selfconsistent solution of the Poisson equation.

In the presence of long range interactions, interacting edge states form a correlated Luttinger liquid. Renormalisation due to edge plasmon excitations enhances the interedge scattering amplitude [39]. This results in a decrease of the localization length of the edge states. Accordingly, the width of the plateaus of lower Landau bands is expected to be reduced due to the Luttinger liquid correlations. Similarly, in the fractional quantum Hall regime, where the edge state excitations are strongly correlated even without long range interactions [39], a reduction of the localization length of edge states is expected as function of the filling factor v.

In 3D layered systems in perpendicular magnetic field, surface states form 2D chiral metals in plateau regions where bulk states are localized [18]. There are transitions between these 2D chiral metals and insulating states in long quasi-1D wires of layered electron systems.

We conclude, that in quantum Hall bars of finite width $L_y \ll \xi_n$ at low temperatures quantum phase transitions occur between extended chiral edge states and a quasi-1D insulator. These are driven by the crossover from 2D to 1D localization of bulk states. These metal-insulator transitions *resemble* first-order phase transitions in the sense that the localization length abruptly jumps between exponentially large and finite values. In the thermodynamic limit, *fixing the aspect ratio* $c = L/L_y$*, when sending* $L \to \infty$*, then* $c \to \infty$*, the two-terminal conductance jumps between exactly integer values and zero.* The transitions occur at energies where the localization length of bulk states is equal to the geometrical wire width. Then, m edge states mix and electrons are free to diffuse between the wire boundaries but become Anderson localized along the wire. At finite temperature, this phenomenon can be observed, when the phase coherence length exceeds the quasi-1D localization length in centers of Landau bands, $L_\varphi > \xi_n$. It may accordingly be called *mesoscopic Chiral Metal-Insulator Transition.* In Hall bars of large aspect ratios at low temperatures one should observe transitions of the two-terminal resistance from integer quantized plateaus, $R_n = h/ne^2$ to a Mott variable-range hopping regime of exponentially diverging resistance. Such experiments would yield new information about edge states in quantum Hall bars. At higher temperature, when $L_\varphi < \xi_n$, the conventional form of the integer quantum Hall effect is recovered [1].

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