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Vector meson electroproduction at next-to-leading order

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The process of a light neutral vector meson electroproduction is studied in the framework of QCD factorization in which the amplitude factorizes in a convolution of the nonperturbative meson distribution amplitude and the generalized parton densities with the perturbatively calculable hard-scattering amplitudes. We derive a complete set of hard-scattering amplitudes at next-to-leading order (NLO) for the production of vector mesons, $V = \rho^0, \omega, \phi$.

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1. The process of elastic neutral vector meson electroproduction on a nucleon,

$$\gamma^*(q)\, N(p) \to V(q')\, N(p'), \quad V = \rho^0,\, \omega,\, \phi\,, \qquad (1)$$

was studied in fix target and in HERA collider experiments. The primary motivation for the strong interest in this process (and in the similar process of heavy quarkonium production) is that it can potentially serve to constrain the gluon density in a nucleon [1, 2]. On the theoretical side, the large negative virtuality of the photon, $q^2 = -Q^2$, provides a hard scale for the process which justifies the application of QCD factorization methods that allow to separate the contributions to the amplitude coming from different scales. The factorization theorem [3] states that in a scaling limit, $Q^2 \to \infty$ and $x_{Bj} = Q^2/2(p \cdot q)$ fixed, a vector meson is produced in the longitudinally polarized state by the longitudinally polarized photon and that the amplitude of the process (1) is given by a convolution of the nonperturbative meson distribution amplitude (DA) and the generalized parton densities (GPDs) with the perturbatively calculable hard-scattering amplitudes. In this contribution we present the results of our calculation of the hard-scattering amplitudes at NLO.

2. $p^2 = p'^2 = m_N^2$ and $q'^2 = m_M^2$, where m_N and m_M are a proton mass and a meson mass respectively. The invariant c.m. energy $s_{\gamma^* p} = (q+p)^2 = W^2$. We define

$$\Delta = p' - p, \quad P = \frac{p+p'}{2}, \quad t = \Delta^2,$$
$$(q-\Delta)^2 = m_M^2, \quad x_{Bj} = \frac{Q^2}{W^2+Q^2}. \qquad (2)$$

We introduce two light-cone vectors

$$n_+^2 = n_-^2 = 0, \quad n_+ n_- = 1. \qquad (3)$$

Any vector a is decomposed as

$$a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_\perp, \quad a^2 = 2\, a^+ a^- - \mathbf{a}^2. \qquad (4)$$

We choose the light-cone vectors in such a way that

$$p = (1+\xi) W n_+ + \frac{m_N^2}{2(1+\xi)W}\, n_-,$$
$$p' = (1-\xi) W n_+ + \frac{(m_N^2 + \mathbf{\Delta}^2)}{2(1-\xi)W}\, n_- + \Delta_\perp. \qquad (5)$$

We are interested in the kinematic region where the invariant transferred momentum, t, is small, much smaller than Q^2. In the scaling limit variable ξ which parametrizes the plus component of the momentum transfer equals $\xi = x_{Bj}/(2 - x_{Bj})$.

GPDs are defined as the matrix element of the light-cone quark and gluon operators [4]:

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$$F^q(x,\xi,t) = = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix(Pz)} \langle p'|\bar{q}\left(-\frac{z}{2}\right) \not{n}_- q\left(\frac{z}{2}\right)|p\rangle|_{z=\lambda n_-} = = \frac{1}{2(Pn_-)}\Big[\mathcal{H}^q(x,\xi,t)\,\bar{u}(p')\,\not{n}_- u(p) + +\mathcal{E}^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{\alpha\beta}n_{-\alpha}\Delta_\beta}{2\,m_N}u(p)\Big], \quad (6)$$

$$F^g(x,\xi,t) = \frac{1}{(Pn_-)}\int\frac{d\lambda}{2\pi}e^{ix(Pz)}\,n_{-\alpha}n_{-\beta}\times \langle\times p'|G^{\alpha\mu}\left(-\frac{z}{2}\right)G^{\beta}_{\mu}\left(\frac{z}{2}\right)|p\rangle|_{z=\lambda n_-} = = \frac{1}{2(Pn_-)}\Big[\mathcal{H}^g(x,\xi,t)\,\bar{u}(p')\,\not{n}_- u(p) + +\mathcal{E}^g(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{\alpha\beta}n_{-\alpha}\Delta_\beta}{2\,m_N}u(p)\Big]. \quad (7)$$

In both cases the insertion of the path-ordered gauge factor between the field operators is implied. Momentum fraction x, $-1\le x\le 1$, parametrizes parton momenta with respect to the symmetric momentum $P=(p+p')/2$. In the forward limit, $p'=p$, the contributions proportional to the functions $\mathcal{E}^q(x,\xi,t)$ and $\mathcal{E}^g(x,\xi,t)$ vanish, the distributions $\mathcal{H}^q(x,\xi,t)$ and $\mathcal{H}^g(x,\xi,t)$ reduce to the ordinary quark and gluon densities:

$$\mathcal{H}^q(x,0,0)=q(x) \quad \text{for } x>0,$$
$$\mathcal{H}^q(x,0,0)=-\bar{q}(-x) \quad \text{for } x<0, \quad (8)$$
$$\mathcal{H}^g(x,0,0)=x\,g(x) \quad \text{for } x>0.$$

Note that gluon GPD is even function of x, $\mathcal{H}^g(x,\xi,t) = = \mathcal{H}^g(-x,\xi,t)$.

The meson DA $\phi_V(z)$ is defined by the following relation

$$\langle 0|\bar{q}(y)\gamma_\mu q(-y)|V_L(p)\rangle_{y^2\to 0} = = p_\mu\, f_V\int_0^1 dz\, e^{i(2z-1)(py)}\phi_V(z). \quad (9)$$

It is normalized to unity $\int_0^1 \phi_V(z)dz = 1$. Here z is a light-cone fraction of a quark, f_V is a meson dimensional coupling constant known from $V\to e^+e^-$ decay, in particular, $f_\rho = 198\pm 7\,\text{MeV}$:

$$\mathcal{M}_{\gamma^*_L N\to V_L N} = \frac{2\pi\sqrt{4\pi\alpha}\,f_V}{N_c\,Q\,\xi}\int_0^1 dz\,\phi_V(z)\int_{-1}^1 dx\,\times \Big[Q_V\left(T_g(z,x)F^g(x,\xi,t)+T_{(+)}(z,x)F^{(+)}(x,\xi,t)\right) + +\sum_q e_q^V T_q(z,x)F^{q(+)}(x,\xi,t)\Big]. \quad (10)$$

Here the dependence of the GPDs, DA and the hard-scattering amplitudes on factorization scale μ_F is suppressed for shortness. Since we consider the leading helicity non-flip amplitude, in eq. (10) the hard-scattering amplitudes do not depend on t. The account of this dependence would lead to the power suppressed, $\sim t/Q$, contribution[3]. α is a fine structure constant, $N_c=3$ is the number of QCD colors. Q_V depends on the meson flavor content. If one assumes it is $\frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$, $\frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ and $|s\bar{s}\rangle$ for ρ, ω and ϕ respectively, than $Q_\rho = 1/\sqrt{2}$, $Q_\omega = 1/3\sqrt{2}$ and $Q_\phi=-1/3$, the sum in the last term of (10) runs over $q=u,d$ for $V=\rho,\omega$ and $q=s$ for $V=\phi$ and

$$e_u^\rho = e_u^\omega = \frac{2}{3\sqrt{2}}, \quad e_d^\rho = -e_d^\omega = \frac{1}{3\sqrt{2}}, \quad e_s^\phi = \frac{-1}{3};$$

$F^{q(+)}(x,\xi,t) = F^q(x,\xi,t) - F^q(-x,\xi,t)$ denotes a singlet quark GPD, $F^{(+)}(x,\xi,t) = \sum_{q=u,d,s}F^{q(+)}(x,\xi,t)$ stands for the sum of all light flavors.

Due to odd C-parity of a vector meson $\phi_V(z) = = \phi_V(1-z)$. Moreover, since V and γ^* have the same C-parities, $\gamma^*\to V$ transition selects the C-even gluon and singlet quark contributions, whereas the C-odd quark combination $F^{q(-)}(x,\xi,t) = F^q(x,\xi,t) + + F^q(-x,\xi,t)$ decouples in (10).

3. Below we present the results of our calculation of the hard-scattering amplitudes in the $\overline{\text{MS}}$ scheme.

$T_q(z,x)$ may be obtained by the following substitution from the known results for a pion EM formfactor, see also [8],

$$T_q(z,x) = = \left\{T\left(z,\frac{x+\xi}{2\xi}-i\epsilon\right) - T\left(\bar{z},\frac{\xi-x}{2\xi}-i\epsilon\right)\right\} + +\left\{z\to\bar{z}\right\}, \quad (11)$$

[3] For the analysis of other helicity amplitudes see [5–7].

$$T(v,u)=\frac{\alpha_S(\mu_R^2)C_F}{4vu}\left(1+\frac{\alpha_S(\mu_R^2)}{4\pi}\times\right.$$
$$\times\left[c_1\left(2[3+\ln(vu)]\ln\left(\frac{Q^2}{\mu_F^2}\right)+\ln^2(vu)+\right.\right.$$
$$\left.+6\ln(vu)-\frac{\ln(v)}{\bar v}-\frac{\ln(u)}{\bar u}-\frac{28}{3}\right)+$$
$$+\beta_0\left(\frac{5}{3}-\ln(vu)-\ln\left(\frac{Q^2}{\mu_R^2}\right)\right)+$$
$$+c_2\left(2\frac{(\bar v v^2+\bar u u^2)}{(v-u)^3}\left[Li_2(\bar u)-Li_2(\bar v)+Li_2(v)-\right.\right.$$
$$\left.-Li_2(u)+\ln(\bar v)\ln(u)-\ln(\bar u)\ln(v)\right]+$$
$$+2\frac{(v+u-2vu)\ln\bar v\bar u}{(v-u)^2}+2[Li_2(\bar u)+Li_2(\bar v)-$$
$$-Li_2(u)-Li_2(v)+\ln(\bar v)\ln(u)+\ln(\bar u)\ln(v)]+$$
$$\left.\left.\left.+4\frac{vu\ln(vu)}{(v-u)^2}-4\ln(\bar v)\ln(\bar u)-\frac{20}{3}\right)\right]\right). \quad (12)$$

Here and below we use a shorthand notation $\bar u = 1-u$ for any light-cone fraction. μ_R is a renormalization scale for a strong coupling, $\beta_0 = 11N_c/3 - 2n_f/3$, n_f is the effective number of quark flavors.

$$C_F=\frac{N_c^2-1}{2N_c},\quad Li_2(z)=-\int_0^z\frac{dt}{t}\ln(1-t).$$

Also we denote

$$c_1=C_F,\quad c_2=C_F-\frac{C_A}{2}=-\frac{1}{2N_c}; \quad (13)$$

$T_{(+)}(z,x)$ starts from NLO

$$T_{(+)}(z,x)=\frac{\alpha_S^2(\mu_R^2)\,C_F}{(8\pi)z\bar z}\,\mathcal{I}_q\left(z,\frac{x-\xi}{2\xi}+i\epsilon\right), \quad (14)$$

here

$$\mathcal{I}_q(z,y)=\left\{\frac{2y+1}{y(y+1)}\left[\frac{y}{2}\ln^2(-y)-\frac{y+1}{2}\ln^2(y+1)\right.\right.+$$
$$\left.+\left(\ln\left(\frac{Q^2z}{\mu_F^2}\right)-1\right)\Big(y\ln(-y)-(y+1)\ln(y+1)\Big)\right]+$$
$$+\frac{y\ln(-y)+(y+1)\ln(y+1)}{y(y+1)}-\frac{R(z,y)}{y+z}+$$
$$\left.+\frac{y(y+1)+(y+z)^2}{(y+z)^2}H(z,y)\right\}+\Big\{z\to\bar z\Big\}, \quad (15)$$

where we introduced two auxiliary functions

$$R(z,y)=z\ln(-y)+\bar z\ln(y+1)+z\ln(z)+\bar z\ln(\bar z), \quad (16)$$

$$H(z,y)=Li_2(y+1)-Li_2(-y)+Li_2(z)-Li_2(\bar z)+\ln(-y)\,\ln(\bar z)-\ln(y+1)\,\ln(z). \quad (17)$$

For the gluonic contribution we obtain

$$T_g(z,x)=\frac{\alpha_S(\mu_R^2)\,\xi}{z\bar z\,(x+\xi-i\epsilon)(x-\xi+i\epsilon)}\times$$
$$\times\left[1+\frac{\alpha_S(\mu_R^2)}{4\pi}\,\mathcal{I}_g\left(z,\frac{x-\xi}{2\xi}+i\epsilon\right)\right], \quad (18)$$

where

$$\mathcal{I}_g(z,y)=\left\{\left(\ln\left(\frac{Q^2}{\mu_F^2}\right)-1\right)\left[c_1\left(\frac{3}{2}+2z\ln(\bar z)\right)\right.\right.+$$
$$+\frac{\beta_0}{2}-\frac{2(c_1-c_2)(y^2+(y+1)^2)}{y(y+1)}\Big((y+1)\ln(y+1)-$$
$$\left.-y\ln(-y)\Big)+\frac{c_1}{2}\left(\frac{y\ln(-y)}{y+1}+\frac{(y+1)\ln(y+1)}{y}\right)\right]-$$
$$-\frac{\beta_0}{2}\left(\ln\left(\frac{Q^2}{\mu_R^2}\right)-1\right)-\frac{c_1(2y+1)R(z,y)}{2(y+z)}-2c_1-$$
$$-\frac{(3c_1-4c_2)}{4}\left(\frac{y\ln^2(-y)}{y+1}+\frac{(y+1)\ln^2(y+1)}{y}\right)+$$
$$+\Big(\ln(-y)+\ln(y+1)\Big)\left[c_1\left(\bar z\ln(z)-\frac{1}{4}\right)+2c_2\right]+$$
$$+c_1(1+3z)\ln(\bar z)-(c_1-c_2)\Big(\ln(z\bar z)-2\Big)\left[\frac{y\ln(-y)}{y+1}+\right.$$
$$\left.+\frac{(y+1)\ln(y+1)}{y}\right]+(c_1-c_2)(2y+1)\ln\left(\frac{-y}{y+1}\right)\times$$
$$\times\left[\frac{3}{2}+\ln(z\bar z)+\ln(-y)+\ln(y+1)\right]+c_1z\ln^2(\bar z)+$$
$$+\Big(c_1(y(y+1)+(y+z)^2)-c_2(2y+1)(y+z)\Big)\times$$
$$\times\left[-\frac{R(z,y)}{(y+z)^2}+\frac{\ln(-y)-\ln(y+1)+\ln(z)-\ln(\bar z)}{2(y+z)}+\right.$$
$$\left.\left.+\frac{y(y+1)+(y+z)^2}{(y+z)^3}\,H(z,y)\right]\right\}+\Big\{z\to\bar z\Big\}. \quad (19)$$

4. Above formulas and the known NLO evolution equations describing μ_F dependence of the GPDs and meson DA give a complete basis for the description of a neutral vector meson electroproduction with NLO accuracy. At leading order we reproduce the known result [9], our results for the NLO hard-scattering amplitudes are new.

At high energies, $W^2 \gg Q^2$, the imaginary part of the amplitude dominates. Leading contribution to the NLO correction comes from the integration region $\xi \ll |x| \ll 1$, simplifying the gluon hard-scattering amplitude in this limit we obtain the estimate

$$\mathcal{M}_{\gamma_L^* N\to V_L N}\approx\frac{-2\,i\,\pi^2\sqrt{4\pi\alpha}\,\alpha_S f_V Q_V}{N_c\,Q\,\xi}\int_0^1\frac{dz\,\phi_V(z)}{z\bar z}\times$$
$$\times\left[F^g(\xi,\xi,t)+\frac{\alpha_S N_c}{\pi}\ln\left(\frac{Q^2 z\bar z}{\mu_F^2}\right)\int_\xi^1\frac{dx}{x}F^g(x,\xi,t)\right]. \quad (20)$$

Given the behavior of the gluon GPD at small x, $F^g(x,\xi,t) \sim$const, we see that NLO correction is parametrically large, $\sim \ln(1/\xi)$, and negative unless one chooses the value of the factorization scale sufficiently lower than the kinematic scale. For the asymptotic form of meson DA, $\phi_V^{as}(z) = 6z\bar{z}$, the last term in (20) changes the sign at $\mu_F = Q/e$, for the DA with a more broad shape this happens at even lower values of μ_F.

It is interesting to note that the value $\mu_F^2 = Q^2/e^2$ is rather close to an estimate in the dipole approach [10] of a typical inverse dipole size [11, 12], $1/r^2 \sim \mu_F^2 = 0.15Q^2$, for vector meson electroproduction in the HERA kinematic region. We believe that a study of relationship between the collinear factorization and the dipole approach at the NLO level would be very important. But this question, as well as the resummation of large at high energies, $\sim (\alpha_S \ln(1/\xi))^n$, contributions both to the hard-scattering amplitudes and to the evolution of GPDs, go beyond the scope of the present work.

5. As an example of our results we compare a one point for the longitudinal cross section reported by ZEUS collaboration [13], e.g. $d\sigma_L/dt|_{t=0} = 17 \pm 4$ nb/GeV^2 at $Q^2 = 27$ GeV^2, $W = 110$ GeV, with our predictions. On the Fig. 1 we plot the dependence of predicted $d\sigma_L/dt|_{t=0}(\mu_F,\mu_R)$ on the factorization scale μ_F for two choices of the renormalization scale: $\mu_R = \mu_F$ (the solid curves) and $\mu_R = Q/\sqrt{e}$, i.e. the BLM (Brodsky-Lepage-McKenzie) prescription (the dashed curves). The data point is described in this plot by the black horizontal line. In this numerical analysis we use two models of the NLO generalized parton distributions of [14]: first one based on MRST2001 forward distribution (the curves a and c) and second one – on CTEQ6M (the curves b and d). Moreover, we take the NLO strong coupling constant α_s and the asymptotic meson DA[4)]. Fig.1 shows that the BLM prescription leads to smaller cross section and to much flatter behaviour of the cross-section on μ_F than for the choice $\mu_R = \mu_F$. We observe also a substantial uncertainty of our predictions due to the input parton GPDs.

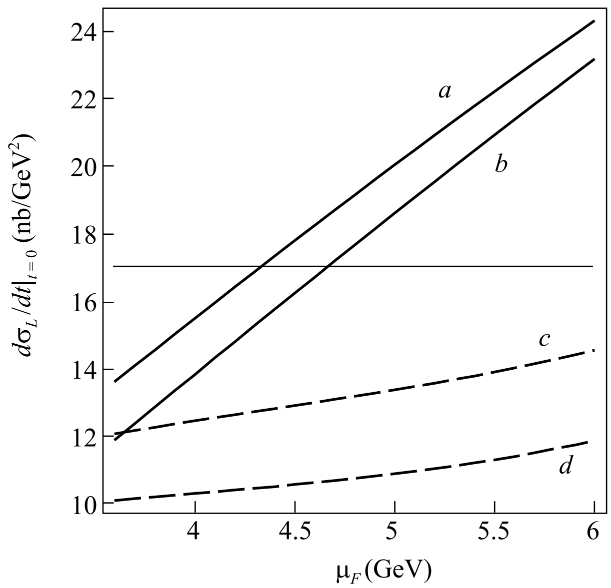

Fig.1. Factorization scale dependence of predicted $d\sigma_L/dt|_{t=0}$ at $Q^2 = 27$ GeV^2, $W = 110$ GeV. The horizontal black line describes the data point. Solid curves assume $\mu_R = \mu_F$, dashed curves – the BLM prescription. The curves a and c (b and d) are obtained with the use of MRST2001 (CTEQ6M)

Since as mentioned above the NLO corrections are large it is instructive to study the relative magnitudes of different contributions to the amplitude (10). This is done by assuming $\mu_R = \mu_F$ and for CTEQ6M GPD. In Fig.2 we show plots of $\mathrm{Im}\,\mathcal{M}$ and $\mathrm{Re}\,\mathcal{M}$ as functions of μ_F corresponding to a gluonic and a quark Born parts of the amplitude (10) (denoted as gB and qB, respectively), to the NLO part of the gluonic contribution T_g (gN), to the NLO part of the quark contribution T_q (qN), to the quark contribution $T_{(+)}$ (q^+) and to the full amplitude (10) (denoted as full). All these separate contributions are normalized by $|\mathcal{M}|$ of (10) with all terms taken into account. Let us note that, as expected for the small x process, the imaginary part of the amplitude (10) dominates over its real part. We observe that the NLO corrections are mostly of opposite signs than the corresponding Born terms and they are big, consequently the final values of the amplitude (10) are the result of a strong cancellations between Born parts and NLO terms. Without account of NLO terms the predictions would be substantially above the data. These results are similar to ones obtained recently for Υ photoproduction [16].

In this paper we restrict our analysis to the leading twist and neglect completely the power suppressed, $\sim 1/Q$, contributions. This is definitely legitimate for sufficiently large Q. Most probably, at the HERA energies and $Q^2 \sim 20\,\mathrm{GeV}^2$ the higher twist corrections are still large, see e.g. [11, 12] based on [17]. Nevertheless, we want to stress that our leading twist results obtained with NLO hard-scattering amplitudes and NLO GPDs [14] (which were adjusted to describe HERA deeply virtual Compton scattering data) are in qualitative agree-

4) QCD sum rules studies [15] show that at low scale the shape of vector meson DA is close to the asymptotic one. At $\mu_F \to \infty$ any DA aproaches the limit – asymptotic DA. Due to this we use asymptotic DA, and postponed the study of dependence on the DA shape for the future analysis.

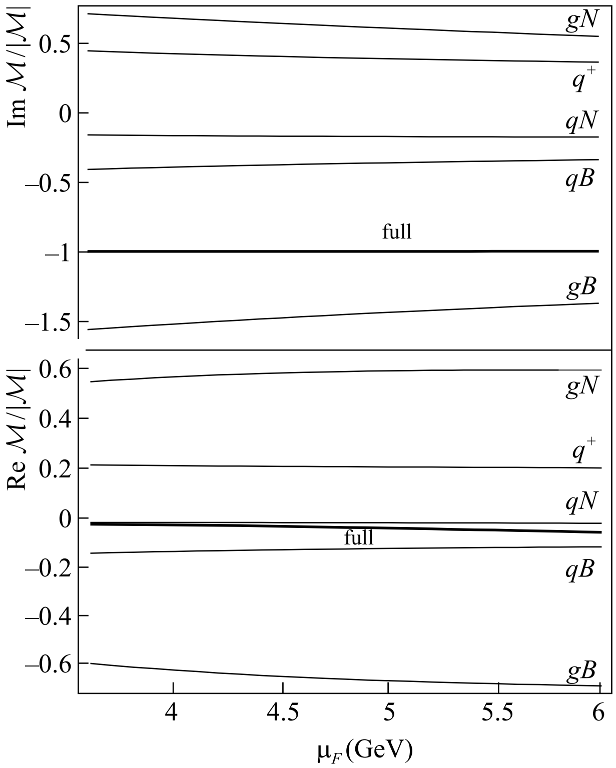

Fig.2. Different contributions to Im$\mathcal{M}/|\mathcal{M}|$ and Re$\mathcal{M}/|\mathcal{M}|$ in dependence on μ_F (for $\mu_R = \mu_F$ and CTEQ6M): gB – Born term of T_g, qB – Born term of T_q, gN – NLO terms of T_g, qN – NLO terms of T_q, q^+ – $T_{(+)}$, full – all terms of (10) included

ment with the measured at HERA ρ meson electroproduction cross section.

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