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Two roads to antispacetime in distorted B-phase of ^{3}He

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The topological materials with emergent analogs of gravity demonstrate the possibility of realization of different exotic spacetimes, including the transition to antispacetime, see, e.g., [1] and references therein. There are several routes to the effective gravity. One of them is the tetrad gravity emerging in the vicinity the Weyl or Dirac points [2–5] – the exceptional crossing points in the fermionic spectrum [6, 7]. Also the degenerate $2+1$ gravity emerges near the Dirac nodal line in the spectrum [1]. Another important source of gravity is the formation of the tetrads as bilinear combinations of the fermionic fields [8–10].

Emergent gravity provides different types of the antispacetime obtained by the space reversal P and time reversal T operations, including those where the determinant of the tetrads e changes sign [9–12]. In cosmology, the antispacetime Universe was in particular suggested as analytic continuation of our Universe across the Big Bang singularity [13]. There were speculations, that antispacetime may support nonequilibrium states with negative temperature as a result of analytic continuation across the singularity [14, 15]. Here we consider the antispacetime realized in experiments [16] on the analog of cosmological walls bounded by strings [17] – Kibble walls (KWs).

In notations [18] used in [19], the Green's function of the relativistic massive Dirac particle has the form:

$$S = \frac{Z(p^2)}{-i\gamma^a e^\mu_a p_\mu + M(p^2)}. \tag{1}$$

Here e^μ_a are tetrads with $\mu, a = 0,1,2,3$; the residue $Z(p^2)$ and the mass $M(p^2)$ are the functions of $p^2 = g^{\mu\nu}p_\mu p_\nu$, where $g^{\mu\nu} = e^\mu_a e^\nu_b \eta^{ab}$. It is convenient to express γ-matrices in terms of two sets of Pauli matrices: σ^1, σ^2 and σ^3 for conventional spin, and τ_1, τ_2, τ_3 for the isospin in the left-right space:

$$\gamma^0 = -i\tau_1, \quad \gamma^a = \tau_2\sigma^a, \quad a = (1,2,3). \tag{2}$$

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \tau_3. \tag{3}$$

In some phases of superfluid ^{3}He, the Green's function for fermionic Bogoliubov quasiparticles is similar to that in Eq. (1). Now instead of the mass function $M(p^2)$, the energy of quasiparticles in the normal Fermi liquid enters, $\epsilon(\mathbf{p}) = v_{\rm F}(|\mathbf{p}| - p_{\rm F})$. The spin matrices σ^a act in the spin space of ^{3}He atoms; the matrices τ_b act in the isotopic Bogoliubov–Nambu space. The function Z can be ignored. The tetrads come from the spin-triplet p-wave order parameter in ^{3}He superfluids – the 3×3 matrix A^i_a with spin index $a = (1,2,3)$ and orbital index $i = (1,2,3)$: $\sum_{\mathbf{k}} k^i \langle a_{\mathbf{k}\alpha} a_{-\mathbf{k}\beta}\rangle \sim A^i_a \left(\sigma^a\sigma^2\right)_{\alpha\beta}$. For the time reversal symmetric phases [20]:

$$A^i_a = p_{\rm F} e^{i\Phi} e^i_a \;, \; a, i = (1,2,3). \tag{4}$$

The tetrads e^i_a emerge due to the spontaneously broken symmetries $SO(3)_S \times SO(3)_L$ under spin and orbital rotations. This is analogous to the formation of the tetrads in relativistic theories as bilinear combinations of the fermionic fields [9, 10]. In addition to tetrads, the order parameter (4) contains the phase Φ coming from spontaneous breaking of $U(1)$-symmetry, and the Green's function depends both on e^μ_a and on Φ:

$$\tilde{S}(e^\mu_a, \Phi) = e^{-\gamma_0\Phi/2} S(e^\mu_a) e^{\gamma_0\Phi/2}. \tag{5}$$

For $\Phi = \pi$ the symmetry transformation $e^{-\gamma_0\Phi/2}$ is equivalent to the conventional space reversal transformation – the parity $P = e^{-\gamma_0\pi/2} = \gamma_0$, with $P^2 = -1$. This suggests that in relativistic theories the discrete symmetry, such as the space inversion P, could be the residual Z_2 symmetry after breaking of the more fundamental symmetry group.

In the time reversal symmetric states realized in experiments [16, 21] the tetrads are:

$$e^i_a = c_1 \hat{\mathbf{f}}_a \hat{\mathbf{x}}^i + c_2 \hat{\mathbf{g}}_a \hat{\mathbf{y}}^i + c_3 \hat{\mathbf{d}}_a \hat{\mathbf{z}}^i, \quad (a,i) = (1,2,3), \tag{6}$$

where $\hat{\mathbf{d}}$, $\hat{\mathbf{f}}$ and $\hat{\mathbf{g}}$ are orthogonal unit vectors in spin space; $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are orthogonal unit vectors in orbital space; c_1, c_2 and c_3 are "speeds of light". In the

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pure B-phase $|c_1| = |c_2| = |c_3|$; in the polar phase [21] $c_1 = c_2 = 0$; in the polar disorted B phase (PdB) $|c_2| = |c_1| < |c_3|$. The particular states of these phases:

$$e_a^\mu = \mathrm{diag}(-1, c_1, c_2, c_3). \tag{7}$$

In the PdB phase, the states with $c_2 = +c_1$ and $c_2 = -c_1$ in Eq. (7) can be separated by the nontopological domain wall – the analog of the KW bounded by strings [17]. The KW typically appears in the two phase transitions: at first transition the linear defect becomes topologically stable; at the second transition the linear defect looses its topological stability and becomes the termination line of the KW. In superfluid ^{3}He, the HQVs (half-quantum vortex – HQV) appear at first transition from the mormal liquid to the polar phase [22], and at further transition to the PdB phase they become the end lines of the KWs [16]. Across KW, $e_2^2 = c_2$ changes sign, and the spacetime analytically transforms to the antispacetime. The intermediate state within the KW has the degenerate tetrad $e_a^\mu = \mathrm{diag}(-1, c_1, 0, c_3)$ – the distorted planar phase (for planar phase $|c_1| = |c_3|$ and $c_2 = 0$ [20]).

Figure 1 demonstrates the loop of HQV, which terminates the KW. In cosmology, the HQV corresponds

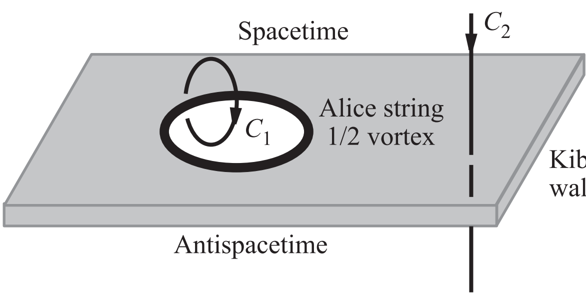

Fig. 1. Roads to antispacetime: the safe route around the Alice string (along C_1) or dangerous route along C_2 across the Kibble wall (through the Alice looking glass)

to the Alice string [23]: by circling around the HQV the phase Φ changes by π, the vectors $\hat{\mathbf{d}}$ and $\hat{\mathbf{f}}$ rotate by π, and one continuously arrives at opposite e_2^2:

$$\mathrm{diag}(-1, c_1, c_2, c_3) \to \mathrm{diag}(-1, c_1, -c_2, c_3), \tag{8}$$

i.e., to the same antispacetime as across the KW.

In conclusion, in the polar distorted B-phase of superfluid ^{3}He, the half-quantum vortex (Alice string) and the Kibble wall bounded by strings demonstrate the two ways to enter the mirror world in Fig. 1: either to go around the HQV or to cross the Kibble wall. The polar distorted B-phase also suggests the scenario of the formation of the discrete symmetry – the parity P in particle physics – from the continuous symmetry existing on the more fundamental level.

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