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Connection formulas between the different QCD orders in application to the valence parton distribution functions

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The formulas directly connecting parton distribution functions (PDFs) at the leading (LO) and next to leading (NLO) QCD orders are applied with respect to both unpolarized and polarized valence PDFs. It is shown that the connection formulas allow without any restriction on the allowed Q^2 range for the analyzed data to produce improved LO results on valence PDFs, which strongly differ from the standard parametrizations on these quantities, and which could be obtained within the standard approach only by using the data produced at very high Q^2 values (that is hardly possible in reality).

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QCD analysis of the experimental data at the leading order (LO) is a very useful tool for the understanding of parton structure of nucleon and fragmentation processes in high energy physics. One of the main advantages of LO is that in contrast to parton distribution functions (PDFs) and fragmentation functions (FFs) at higher QCD orders these quantities at LO not only can be used as an input for calculation (in the chosen factorization scheme) of asymmetries and cross sections of different processes, but being scheme independent have the physical meaning themselves, allowing the probabilistic interpretation and, thereby, the possibility of qualitative physical predictions. That is why LO analysis is very needed even today and many modern collaborations produce namely LO results on PDFs/FFs (see, for instance, [1, 2]). At the same time, as we will see below, the inclusion of the data produced at small and moderate Q^2 in the standard LO analysis is really dangerous for the LO results: as we will see below, the difference between the LO results on PDFs with and without inclusion of the small and moderate Q^2 data into analysis is significant and is not covered by the error band. However, now did appear a good possibility to produce the correct LO results on PDFs/FFs without lost of events collected in experiment at small and moderate Q^2, which make up the overwhelming majority of events available to any experiment.

Recently [3] the formulas directly connecting parton distribution functions and fragmentation functions at the next to leading order (NLO) of QCD with the same quantities at the leading order were derived (see Eqs. (15), (16) in Ref. [3]). To obtain these formulas only the DGLAP evolution equations and the asymptotic condition that PDFs (FFs) at different QCD orders become the same in the Bjorken limit were used as an input. Due to universality of this input the obtained connection formulas are also universal, i.e. they are valid for any kind of PDFs (FFs) we deal with, differing only in the respective splitting functions entering there. Moreover, operating in the same way as in Ref. [3] one can also establish the connection of PDFs (FFs) at LO (as well as at the NLO) with these quantities at any higher QCD order (NNLO, NNNLO, ...) (will be published elsewhere).

In this paper we focus on the (both unpolarized and polarized) valence PDFs. The non-singlet connection formulas, Eq. (16) in Ref. [3], for the unpolarized $q_V \equiv q - \bar{q}$ and polarized $\Delta q_V \equiv \Delta q - \Delta\bar{q}$ (helicity PDFs) valence PDFs look as

$$q_V(Q^2,x) = \\ = \left\{\delta(1-x) + \frac{\alpha_s(Q^2)}{2\pi}\left[\frac{\beta_1}{\beta_0^2}P^{(0)}(x) - \frac{2}{\beta_0}P^{(1)}(x)\right]\right\}\otimes \\ \otimes \mathrm{Exp}\left[-\frac{2}{\beta_0}\ln\frac{\alpha_s(Q^2)}{\hat{\alpha}_s(Q^2)}\,P^{(0)}(x)\right]\otimes \hat{q}_V(Q^2,x), \quad (1)$$

and analogously for the valence helicity PDFs with replacements[2)] $q_V(\hat{q}_V) \to \Delta q_V(\Delta\hat{q}_V)$, $P_V \to \Delta P_V$. Here the notation of Ref. [3]

$$A\big|_{\mathrm{LO}} \equiv \hat{A}, \quad A\big|_{\mathrm{NLO}} \equiv A, \quad (2)$$

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[2)] All over the paper we will use the most often used $\overline{\mathrm{MS}}$ scheme for the quantities at NLO. The respective unpolarized $P_V^{(1)}(x)$ and polarized $\Delta P_V^{(1)}(x)$ splitting functions can be found, for instance, in Refs. [4] and [5].

for any quantity A at LO and NLO is used, the convolution ($\otimes$) is defined in standard way

$$(A\otimes B)(x)=\int_0^1 dx_1\int_0^1 dx_2\delta(x-x_1x_2)A(x_1)B(x_2)= \\ =\int_x^1 \frac{dy}{y}A\left(\frac{x}{y}\right)B(y),$$

while the generalized exponent $\mathrm{Exp}\left[\epsilon P_V^{(0)}(x)\right]$ in Eq. (1) is defined as a series

$$\mathrm{Exp}\left[\epsilon\, P_V^{(0)}(x)\right]=\delta(1-x)+\epsilon P_V^{(0)}(x)+ \\ +(\epsilon^2/2!)\,P_V^{(0)}(x)\otimes P_V^{(0)}(x)+\ldots \quad (3)$$

(analogously for $\mathrm{Exp}\left[\epsilon\Delta P_V^{(0)}(x)\right]$), where the parameter $\epsilon\equiv(-2/\beta_0)\ln(\alpha_s/\hat\alpha_s)$ is very small even at the minimal really available Q^2 values (the lower boundary of the experimental cut on Q^2 is usually about $1\,\mathrm{GeV}^2$), so one can achieve very good accuracy keeping only few first terms in this expansion. Within this paper we will keep only two terms in the expansions like Eq. (3), because $O(\epsilon^2)$ corrections are really negligible at the reference scale $Q^2_{\mathrm{ref}}=10\,\mathrm{GeV}^2$ chosen in the paper.

Let us make an important remark concerning the validity of Eq. (1), which is just the particular realisation of non-singlet connection formula (16) in Ref. [3]. One can be rather confused by the application of evolution up to infinite point and inverse evolution to initial point applied at the derivation of the connection formulas (15), (16) in Ref. [3]. However, of importance is that to understand the correctness of Eqs. (15), (16) in Ref. [3], the history of obtaining these formulas is not especially important, because one can just forget for a moment all delicate moments of their derivation and *directly check* their validity (see footnote 3 in Ref. [3]). In particular, using the obvious relation $Q^2 d\left\{\mathrm{Exp}\left[(2/\beta_0)\ln\hat\alpha_s P_V^{(0)}\right]\otimes\hat q_V\right\}/dQ^2=0$ one can immediately check that r.h.s. of Eq. (1) indeed satisfies the NLO DGLAP evolution equation

$$Q^2\frac{dq_V(Q^2,x)}{dQ^2}= \\ =\frac{\alpha_s}{2\pi}\left[P_V^{(0)}(x)+\tfrac{\alpha_s}{2\pi}P_V^{(1)}(x)+O(\alpha_s^2)\right]\otimes q_V(Q^2,x),\quad (4)$$

if only $\hat q_V$ satisfies LO DGLAP equation $Q^2 d\hat q_V(Q^2,x)/dQ^2=(\alpha_s/2\pi)P_V^{(0)}(x)\otimes\hat q_V(Q^2,x)$. Besides, one can see that q and $\hat q$ connected by Eq. (1) indeed coincide in the Bjorken limit, i.e., satisfy the asymptotic condition Eq. (11) in Ref. [3], which is the key point for the derivation of connection formulas.

Let us now proceed with an *important statement*: applying the connection formulas like Eq. (1) one should avoid the naive substitution of the existing LO parametrizations to r.h.s. of these formulas. The point is that all LO parametrizations of PDFs (FFs) existing today are obtained using the LO theoretical expressions (i.e., excluding all QCD corrections to improved QPM) for the cross sections and asymmetries measured in experiment (actually including *all QCD corrections*) at all, even very small (about 1 GeV^2) available Q^2 values. At the same time, as we will see below, this is dangerous[3)] procedure and leads to the strong distortion of the PDFs behaviour at LO. We will see, that such procedure of LO analysis can produce correct (realistic) LO PDFs (FFs) only in very high Q^2 region and only PDFs (FFs) obtained there should then be evolved via DGLAP equations to the small and moderate Q^2 region. Regretfully, the most of statistics is available just at small and moderate Q^2, so that this way of action seems to be hardly feasible. However, as it was mentioned above, now there is a good possibility to approach the correct PDFs/FFs at LO without any restrictions on Q^2 for the analyzed data. To this end one should just inverse the connection formulas (Eqs. (15), (16) in Ref. [3]), thereby expressing LO quantities through extracted from the data NLO ones. It is clear that, PDFs (FFs) at LO obtained in this way are much more realistic since one believes that NLO theoretical expressions for asymmetries and cross sections well approximate their measured values even at low Q^2. Thus, it seems to be natural to call LO PDFs obtained in this way the "improved LO PDFs". Later we will argue (see a numerical experiment below) that such improved PDFs/FFs actually correspond to the results which could be obtained within the standard procedure of LO analysis by using only the "ideal" data produced at extremely high Q^2 values.

Let us first consider very important case of unpolarized parton densities and obtain the improved nonsinglet unpolarized valence PDFs $\hat q_V\equiv\hat q-\hat{\bar q}$ at LO via substitution of some modern NLO parametrization of q_V into the inverse of Eq. (1) connection formula

$$\hat q_V(Q^2,x)= \\ =\left\{\delta(1-x)-\frac{\alpha_s(Q^2)}{2\pi}\left[\frac{\beta_1}{\beta_0^2}P^{(0)}(x)-\frac{2}{\beta_0}P^{(1)}(x)\right]\right\}\otimes \\ \otimes\,\mathrm{Exp}\left[\frac{2}{\beta_0}\ln\frac{\alpha_s(Q^2)}{\hat\alpha_s(Q^2)}P^{(0)}(x)\right]\otimes q_V(Q^2,x),\quad (5)$$

and then compare the obtained result with the respective original LO parametrization of $\hat q_V$. We choose here the modern and widely used [7] MSTW2008

3) For instance, it is well known that at moderate Q^2 the theoretical values for Drell–Yan cross sections are about two times higher (K-factor) than the respective LO ones [6].

parametrization both for NLO and LO unpolarized PDFs.

Improved MSTW2008 LO parametrization of $\hat{q}_V$ obtained via the inverse connection formula (5) is presented in Fig. 1 in comparison with the original

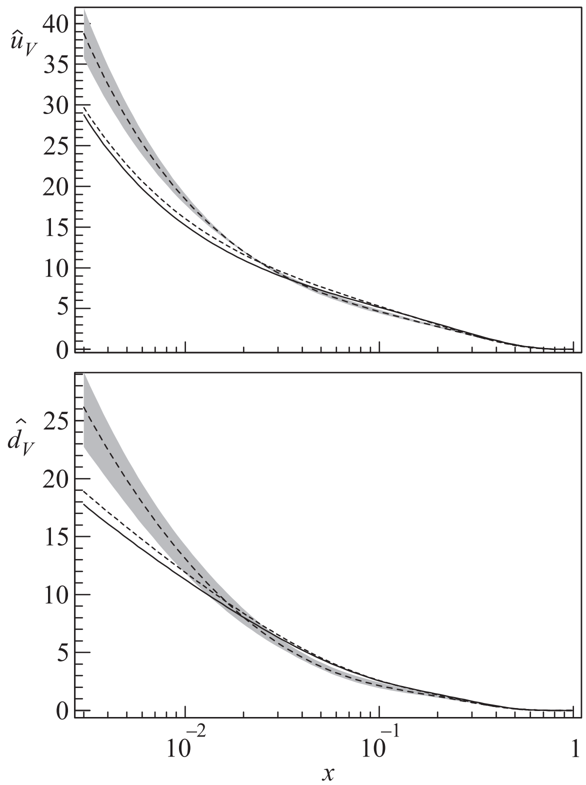

Fig. 1. Results on $\hat{u}_V$ and $\hat{d}_V$ versus x at $Q^2_{\rm ref} = 10$ GeV^2. The dashed line is MSTW2008 LO parametrization [7]. The gray area is the corresponding error band. The bold solid line corresponds to improved MSTW2008LO parametrization of $\hat{u}_V$ and $\hat{d}_V$, which is obtained by using the MSTW2008 NLO parametrization in r.h.s. of inverse connection formula Eq. (5). For comparison, the results of MSTW2008 NLO parametrization on $q_V(10\ \text{GeV}^2, x)$ are also presented here by the bold dashed lines

MSTW2008 LO parametrization of $\hat{q}_V$. One can see that the improved LO parametrization essentially differs from the respective LO parametrization on $\hat{q}_V$ obtained in standard way, and the difference is not covered by error band.

For comparison, the MSTW2008 NLO parametrization of $q_V(10\ \text{GeV}^2, x)$ is also presented in Fig. 1 by the bold dashed line. It seems to be of importance that while it strongly differs from the original LO parametrization MSTW2008 LO, the improved parametrization of $\hat{q}_V$ is very close to the respective original NLO parametrization MSTW2008 NLO on q_V.

To better realize what happens when one performs LO analysis of the data let us perform the *numerical experiment* and prepare the artificial "ideal" polarized semi-inclusive DIS data on pion production. Namely, we consider as such "pseudo-experimental" input the integrated over cut in z proton and deuteron difference asymmetries[4)] $A^{\rm exp}_{p(d)}\big|_Z$, constructed by substitution of the known NLO parametrizations of u_V, d_V, Δu_V, Δd_V and difference of favored and unfavored pion FFs $D_1 - D_2$ into the theoretical expressions for these asymmetries at NLO (see Eqs. (6)–(10) in Ref. [8]). Then, we extract $\Delta\hat{u}_V$ and $\Delta\hat{d}_V$ at LO using these constructed asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ as a "pseudo-experimental" input in the LO equations (all notation is just as in Ref. [8])

$$A^{\rm exp}_p\big|_Z(x, Q^2) = (1+R)\frac{4\Delta\hat{u}_V - \Delta\hat{d}_V}{4\hat{u}_V - \hat{d}_V},$$
$$A^{\rm exp}_d\big|_Z(x, Q^2) = (1+R)\left(1 - \frac{3}{2}\omega_D\right)\frac{\Delta\hat{u}_V + \Delta\hat{d}_V}{\hat{u}_V + \hat{d}_V}, \quad (6)$$

i.e., we perform the LO analysis analogous to one of COMPASS [9], but with the replacement of the real data on $A^{\rm exp}_{p(d)}\big|_Z$ by the prepared pseudo-data on $A^{\rm exp}_{p(d)}\big|_Z$. The obvious advantage of the latter is that they exist in the widest range in Q^2, up to 10^6 GeV^2 in accordance with the upper Q^2 boundary for parametrizations which we use for the pseudo-data preparation. Making use of this advantage we can now comprehensively investigate the influence of Q^2 on the produced with Eqs. (6) LO results. Namely, we now have a possibility to obtain and compare the LO result on $\Delta\hat{q}_V$ at the same reference scale ($Q^2_{\rm ref} = 10$ GeV^2 here) with and without involving the dangerous data at small and moderate Q^2 into the analysis.

The respective results on $\Delta\hat{u}_V$ are presented in Figs. 2 and 3 (the results on $\Delta\hat{d}_V$ are analogous), where, besides the modern parametrization DSSV of Δq (Fig. 3), we use also older GRSV parametrization as an input in Eqs. (6) (Fig. 2), because the latter has the LO version, which is useful for comparison.

Dashed line in Fig. 2 corresponds to the original GRSV LO parametrization, while solid and dotted lines in Figs. 2 and 3 correspond to direct extraction of $\Delta\hat{q}_V$ with Eqs. (6) (direct solution of the system (6) with respect to $\Delta\hat{u}_V$ and $\Delta\hat{d}_V$), where asymmetries $A^{\rm exp}_{p,(d)}\big|_Z$ are calculated substituting GRSV [10] NLO (Fig. 2) or

[4)] These asymmetries are an ideal tool for our research, because in contrast to all other known asymmetries (and cross sections) they contain at LO nothing except the valence PDFs. The similar object $F_2^{\pi^+} - F_2^{\pi^-}$ in unpolarized semi-inclusive DIS is not so convenient for our purposes, because it involves pion FFs already at LO.

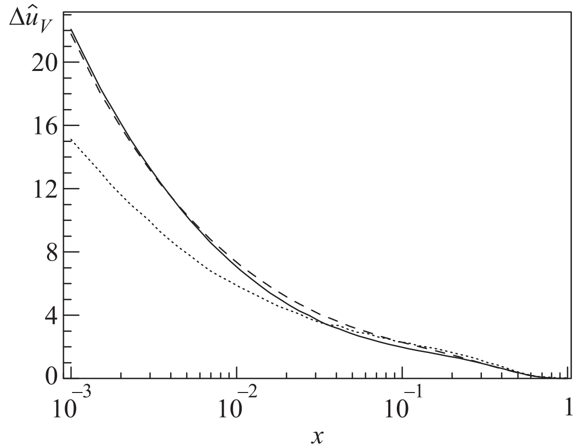

Fig. 2. $\Delta\hat{u}_V$ at $Q^2_{\rm ref} = 10$ GeV^2 versus x. Dashed line corresponds to the original GRSV LO parametrization. Solid line corresponds to direct solution of the system (6) with respect to $\Delta\hat{q}_V$, where asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ are taken at $Q^2_{\rm ref} = 10$ GeV^2 and MSTW2008 LO parametrization of $\hat{q}_V$ is used in denominators. Asymmetries $A^{\rm exp}_{p,(d)}\big|_Z$ are calculated substituting GRSV NLO parametrization of polarized PDFs, MSTW2008 NLO parametrization of unpolarized PDFs, and AKK parametrization of FFs into the respective NLO equations (Eqs. (6)–(10) in Ref. [8]). Dotted line corresponds to $\Delta\hat{q}_V$ which are obtained in the same way but at extremely high $Q^2 = 10^6$ GeV^2 and with improved MSTW2008 LO parametrization of $\hat{q}_V$ (bold solid lines in Fig. 1) in denominators of Eqs. (6). The obtained in this way $\hat{q}_V$ are then evolved via DGLAP equations from $Q^2 = 10^6$ GeV^2 to $Q^2_{\rm ref} = 10$ GeV^2

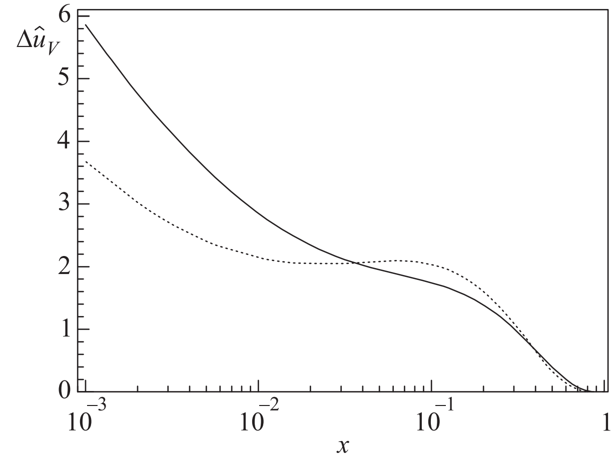

Fig. 3. $\Delta\hat{u}_V$ at $Q^2_{\rm ref} = 10$ GeV^2 versus x. Solid line corresponds to direct solution of the system (6) with respect to $\Delta\hat{q}_V$, where asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ are taken at $Q^2_{\rm ref} = 10$ GeV^2 and MSTW2008 LO parametrization on $\hat{q}_V$ is used in denominators. Asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ are calculated substituting DSSV NLO parametrization of polarized PDFs, MSTW2008 NLO parametrization of unpolarized PDFs, and AKK parametrization of FFs into the respective NLO equations (Eqs. (6)–(10) in Ref. [8]). Dotted line corresponds to $\Delta\hat{q}_V$ which are obtained in the same way but at extremely high $Q^2 = 10^6$ GeV^2 and with improved MSTW2008 LO parametrization of $\hat{q}_V$ (bold solid lines in Fig. 1) in denominators of Eqs. (6). The obtained in this way $\hat{q}_V$ are then evolved via DGLAP equations from $Q^2 = 10^6$ GeV^2 to $Q^2_{\rm ref} = 10$ GeV^2

DSSV [11] NLO (Fig. 3) parametrizations of polarized PDFs, MSTW2008 [7] NLO parametrization of unpolarized PDFs, and AKK [12] parametrization of FFs into the respective NLO equations (Eqs. (6)–(10) in Ref. [8]). At the same time, very important difference between $\Delta\hat{q}_V$ presented by solid and dotted curves is that the first is obtained with asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ in Eqs. (6) taken directly at $Q^2_{\rm ref} = 10$ GeV^2, while the second is obtained with asymmetries $A^{\rm exp}_{p(d)}\big|_Z$ taken at extremely high $Q^2 = 10^6$ GeV^2 and only then are evolved to the reference scale $Q^2_{\rm ref} = 10$ GeV^2. Besides, of importance is that instead of the original MSTW2008 LO parametrization of $\hat{q}_V$ used for construction of solid lines in Figs. 2 and 3, the respective improved LO parametrization (given by bold solid lines in Fig. 1) is used in denominators of Eqs. (6) for construction of the dotted lines. Indeed, it is clear that to produce the completely corrected (improved) LO results on $\Delta\hat{q}_V$ one should completely exclude the influence of small Q^2 on the analysis with Eq. (6). To this end it is not sufficient to restrict the pseudo-data in l.h.s. of Eqs. (6) by the high Q^2 values and one should[5)] also properly correct/improve the denominators in Eqs. (6).

Comparing solid and dashed curves in Fig. 2 one can see that they are very close to each other, which is not surprising, since the vast majority of data used for production of the original GRSV LO parametrization (as well as for production of any other known parametrization) correspond to small and moderate Q^2 values close to 10 GeV^2. In contrast, both of these curves in Fig. 2, as well as solid line in Fig. 3 strongly differ from the respective dotted lines, corresponding to the "pure" LO results on $\Delta\hat{q}_V(10\ {\rm GeV}^2, x)$, which are free from the involvement of small Q^2 data in the analysis.

We will see now that these "pure" LO results (which within the standard approach could be obtained only completely excluding small and moderate Q^2 data from

[5)] Otherwise, one implicitly involves in the LO results on $\Delta\hat{q}_V$ the small and moderate Q^2 data used to produce the original LO parametrization on $\hat{q}_V$ and obtains some intermediate (only "partially improved") results: our calculations show that the respective line lies between dotted and solid (dashed) lines in Fig. 2.

the analysis) can be reproduced by using the connection formula between LO and NLO of QCD.

Thus, let us apply the inverse of Eq. (1) connection formula

$$\Delta\hat{q}_V(Q^2,x) = \\ = \left\{\delta(1-x) - \frac{\alpha_s(Q^2)}{2\pi}\left[\frac{\beta_1}{\beta_0^2}\Delta P^{(0)}(x) - \frac{2}{\beta_0}\Delta P^{(1)}(x)\right]\right\}\otimes \\ \otimes \,\mathrm{Exp}\left[\frac{2}{\beta_0}\ln\frac{\alpha_s(Q^2)}{\hat{\alpha}_s(Q^2)}\Delta P^{(0)}(x)\right]\otimes \Delta q_V(Q^2,x), \quad (7)$$

written in terms of the valence helicity PDFs. Substituting in r.h.s. of (7) the modern DSSV NLO parametrization of Δq_V (used for production of Fig. 3) taken at the reference scale $Q^2_{\mathrm{ref}} = 10$ GeV^2 we obtain the respective LO results on $\Delta\hat{q}_V$ at the same Q^2_{ref}. The results on $\Delta\hat{q}_V(10\ \mathrm{GeV}^2, x)$ obtained in this way are presented in Fig. 4 by the bold solid lines and are compared with the respective results, obtained earlier by the use of Eqs. (6) and presented in Fig. 3. Comparing dotted and bold solid lines one can see that while they strongly differ from the thin solid line directly obtained via Eq. (6) at small $Q^2_{\mathrm{ref}} = 10$ GeV^2, they are in excellent agreement with each other. Moreover, they almost coincide with each other. Thus, one can conclude that to reach improved LO valence PDFs there is no need to restrict the analysis to use of data produced only at extremely high Q^2 values (that is hardly possible in reality): the inverse connection formulas give the direct access to the same improved LO PDFs without any additional[6)] restrictions on the Q^2 range from which the data comes. In contrast, the standard procedure of LO analysis can produce these improved PDFs (FFs) at LO only from data in very high Q^2 region and only after that they should be evolved with DGLAP to the lower Q^2 values. However, regretfully, the most of statistics is available just at small and moderate Q^2, so that the such a way of action is realizable in our simple model but seems to be hardly possible in reality. Looking at Fig. 4 it is possible to draw another important (and rather surprising) conclusion: just as in unpolarized case (see discussion around Fig. 1) the improved LO results (bold solid and dotted lines) are much closer to the respective NLO results (bold dashed line) than to LO results obtained in standard way (thin solid line).

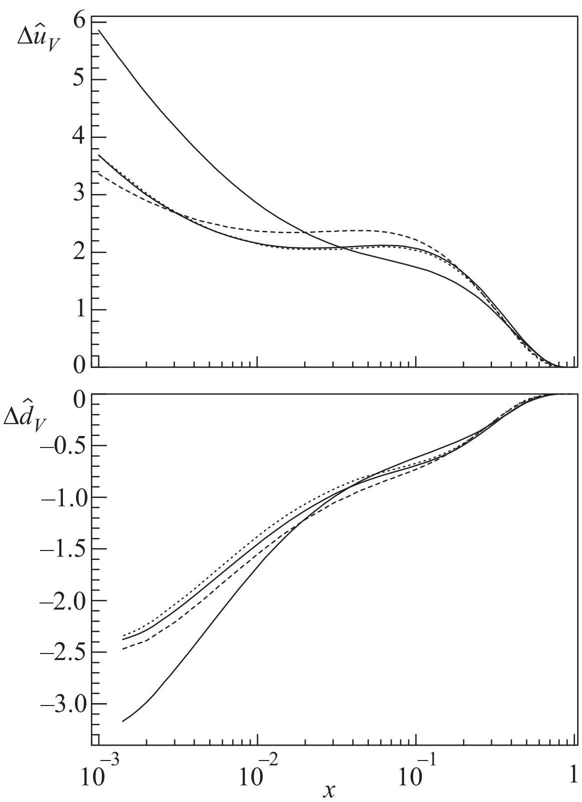

Fig. 4. LO valence helicity PDFs $\Delta\hat{u}_V$ and $\Delta\hat{d}_V$ at $Q^2_{\mathrm{ref}} = 10$ GeV^2 obtained in different ways. Bold solid line corresponds to improved LO parametrization, which is obtained from DSSV NLO parametrization via inverse connection formula (7). Thin solid and dotted lines are just the same as in Fig. 3. For comparison, the results of DSSV NLO parametrization of $\Delta q_V(10\ \mathrm{GeV}^2, x)$ are also presented here by the bold dashed lines

In conclusion, the non singlet (inverse) connection formulas (5), (7) between LO and NLO QCD orders were applied to both unpolarized and polarized valence PDFs. It was shown that the connection formulas produce improved LO results on valence PDFs without exclusion from the analysis the data coming from the small and moderate Q^2 region (only the standard cut $Q^2 > 1$ GeV^2 should be applied). The respective improved LO parametrizations on the valence PDFs were obtained (bold solid lines in Figs. 1 and 4), and they strongly differ from the respective original LO parametrizations, obtained by using the most of data produced at small and moderate Q^2 values. Of importance is the argued statement that within the standard procedures of LO analysis (LO theoretical expressions are applied to the measured asymmetries and cross sections) the same improved LO results could be obtained by using only the data produced at extremely high Q^2 values, that is hardly possible in reality. It is rather amazing that improved LO results (bold solid lines in Figs. 1 and 4) are much closer to the respective NLO results (bold dashed line in Figs. 1 and 4) than to LO results obtained in standard way (thin solid lines in Figs. 1 and 4).

[6)] Only the standard cut $Q^2 > 1$ GeV^2, providing pQCD applicability, should be applied.

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